Reactive Power Planning using Security-Constrained AC Optimal Power Flow and Sensitivity Analyses

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***Abstract*—This paper proposes an approach for siting and sizing reactive power sources in order to maintain adequate reactive power reserves in future loading scenarios. We consider both normal and post-contingency operation using a security-constrained optimal power flow (SCOPF) formulation to compute the minimum-required reactive power reserves. To maintain tractability, the SCOPF explicitly considers a small set of important contingencies that are identified using a continuation power flow. The remaining contingencies are evaluated using the SCOPF solution. Any violated contingencies are iteratively added to the considered set in the SCOPF formulation until all contingencies are satisfied. Our approach combines this SCOPF formulation with a bus sensitivity index that identifies potential locations for new reactive power supply. We demonstrate the proposed approach on the IEEE 30- and 118-bus test cases.**

***Index Terms*—Continuation Power Flow, Reactive Power Planning, Security-Constrained AC Optimal Power Flow.**

I. Introduction

The supply of reactive power is essential for power system operation and voltage stability. Reactive power cannot be transmitted over long distances and must therefore be supplied locally [1]. Many of the large conventional generators that have traditionally provided reactive power support are being retired. Furthermore, Distributed Energy Resources (DERs) are expected to play a larger role in future power systems. DERs are typically operated with a focus on their active power outputs, either due to a lack capabilities or regulatory mandates for their reactive power support [2]. Thus, provision of additional reactive power supply is critical for securely operating future power systems [3]. Reactive power demand is expected to increase in the future as more loads are connected to the system, leading to voltage stability and security concerns.

To ensure stable operation of the power system, operators need to maintain adequate Voltage Stability Margins (VSM) during both the base case and contingencies (i.e., line and generator failure scenarios). The VSM can be interpreted as the difference between the base case loading and the *critical point*, i.e., the maximum loading condition of the system where the power flow Jacobian becomes singular [4]. There is a strong correlation between the VSM and reactive power supply [5]. Increasing the reactive power supply generally enhances the stability margins. System operators typically plan for sufficient Reactive Power Reserve (RPR) to ensure stable operation during contingencies. In addition to the direct loss of reactive power from generator contingencies, the voltage drops that can occur during contingencies reduce the reactive power produced from shunt capacitors and line charging susceptances, causing a further reduction in the reactive power supply [6]. As shown in Fig. 1, the power–voltage curve illustrates the relationship between the VSM and the critical point. During contingencies, the critical point value can change significantly, thus leading to a lower VSM as depicted in Fig. 1. The RPR requirement must be supplied during contingencies by fast reactive power supplies, i.e., generators, synchronous condensers, and Flexible AC Transmission System (FACTS) devices such as STATCOMs [7].

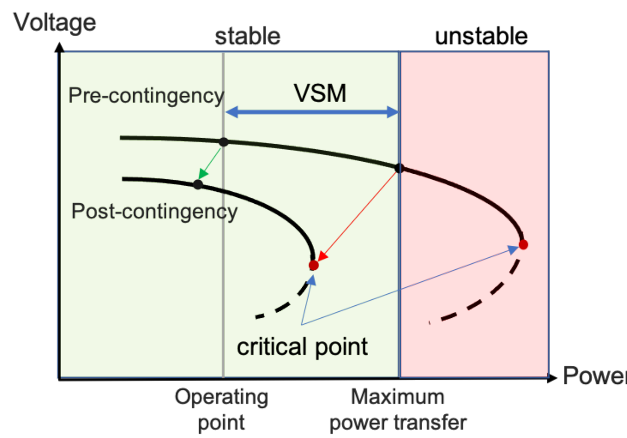

Fig. 1. Active power voltage curve showing the critical point pre- and post-contingency.

Several studies discuss the RPR allocation problem to support power system stability from both operational [6]–[8] and planning [3], [9]–[12] perspectives. From an operational perspective, [6] manages existing reactive power supplies using an optimal power flow formulation. A two-stage bender's decomposition is used in [6], where the first stage consists of the base case and the second stage contains multiple stressed cases. Further, the authors of [6] use a volt-var curve method to predetermine a set of generators to provide RPR support and to define the participation factors. Reference [7] proposes a real-time optimization formulation to find the optimal control action that maximizes a linearized RPR sensitivity index.

In [8], the authors propose a two-stage mixed integer dynamic programming problem to find a day-ahead reactive power dispatch. The first stage solves for the discrete reactive power compensation using a heuristic search, while the second stage solves for the continuous reactive power variables using a variable correction method.

From a planning perspective, the authors of [3] propose using a static sensitivity indices to rank the contingencies and select the candidate buses to be considered in a Security-Constrainted Optimal Power Flow (SCOPF) problem. The indices are computed by comparing the voltage levels in pre- and post-contingency conditions. The author of [9] proposes a modified AC SCOPF problem to find the optimal RPR requirement with two formulations: one for reactive power production and the other for reactive power absorption. In [10], a predictor-corrector optimization method is proposed to find the minimum reactive power requirement. The authors of [10] use a continuation power flow (CPF) algorithm to predict an incremental loading step from the base case. The voltage levels are assessed at each CPF iteration. If the voltage levels violate the limits, an optimization problem is used to allocate additional reactive support. The authors of [11] formulate a reactive power planning problem using an L2-norm regularization and solve the problem using a successive conic programming algorithm with an adaptive trust-region control scheme. Using the L2-norm regularization allows to find a solution with minimum reactive power supply locations. Reference [12] also uses Lq regularization, where q is between 0 and 1, to allocate FACTS devices by solving sparsity-constraints OPF problem.

In this paper, we propose a new methodology for finding the minimum RPR requirement using an SCOPF formulation that includes an L1-norm to reduce the number of locations selected for additional reactive power supplies. We first use a CPF-based method to identify important contingencies that will be explicitly enforced in the SCOPF problem. To find a solution with a low number of additional reactive power supplies, we rank the buses based on their tangent value near the critical point in order to identify candidate locations for additional reactive power supply. We then propose an iterative process which adds reactive power support at these candidate locations and solves an SCOPF problem with the selected contingencies. Using this solution, we update the ranking of candidate locations as well as the set of contingencies that are explicitly enforced in the SCOPF problem and then compute a new solution. This methodology provisions reactive power reserves that ensure feasibility of the system during a future loading scenario.

The main contribution of this paper is our proposed methodology for finding the optimal allocation of RPR requirements using a combination of several well-established concepts in the literature [3], [9], [12]. We use an SCOPF-based method with iterative contingency enforcement to optimally site and size the RPR requirements. We use an L1-norm in the SCOPF objective function to limit the number of additional RPR locations. We also use a CPF method to identify the most severe contingencies and the potential candidate buses for the installation of new reactive power supplies. Our approach differs from [3] as we use the CPF results to determine the bus sensitivities near the critical point, and differs from [10] in that we use the CPF algorithm to rank the buses followed by iteratively solving the SCOPF problem. Explicitly enforcing a subset of the contingencies in this manner provides significant advantages in computational tractability while still ensuring the steady-state voltage stability of a future loading scenario with minimum additional reactive power support.

II. Methodology

The proposed methodology uses an SCOPF formulation to find the optimal RPR siting and sizing. We use an iterative process based on a CPF calculation to select the contingencies that are explicitly enforced in the SCOPF. Further, we use a bus sensitivity analysis to identify potential RPR locations. Section II-A introduces our SCOPF formulation. Section II-B presents the contingency selection criteria and bus sensitivity analysis. Section II-C describes the overall methodology.

A. Reactive Power Reserve using SCOPF

SCOPF plays a major role in power system planning and operation. SCOPF extends the Optimal Power Flow (OPF) problem by accounting for operation during contingencies in addition to nominal operation. When the network is modeled using the AC power flow equations, the SCOPF problem is non-linear, non-convex, and NP-hard [13]. Nevertheless, nonlinear programming algorithms such as interior point methods [14] are often able to find high-quality feasible operating points for SCOPF problems. We formulate an SCOPF problem whose solution yields an optimal allocation of reactive power by minimizing the installation cost of the reactive power supply. We incorporate the RPR in the formulation as the maximum reactive power supply needed for any contingency. The SCOPF formulation is:

$$\min \quad \sum_{i \in \mathcal{G}} f_i(p_i^0) + \sum_{j \in \mathcal{S}} c_R |RPR_j| \tag{1a}$$

s.t.

$$P_{G_i}^c - P_{L_i}^c = \sum_{j \in \mathcal{N}_i^c} P_{ij}^c(V_i^c, V_j^c, \delta_i^c, \delta_j^c), \quad \forall i \in \mathcal{N}, \tag{1b}$$

$$Q_{G_i}^c + Q_{R_i}^c - Q_{L_i}^c = \sum_{j \in \mathcal{N}_i^c} Q_{ij}^c(V_i^c, V_j^c, \delta_i^c, \delta_j^c), \quad \forall i \in \mathcal{N}, \tag{1c}$$

$$|I_{ij}^c(V_i^c, V_j^c, \delta_i^c, \delta_j^c)| \leq I_{ij}^{max}, \quad \forall (i,j) \in \mathcal{T}, \tag{1d}$$

$$V_i^{min} \leq V_i^c \leq V_i^{max}, \quad \forall i \in \mathcal{N}, \tag{1e}$$

$$P_{G_i}^{min} \leq P_{G_i}^c \leq P_{G_i}^{max}, \quad \forall i \in \mathcal{G}, \tag{1f}$$

$$Q_{G_i}^{min} \leq Q_{G_i}^c \leq Q_{G_i}^{max}, \quad \forall i \in \mathcal{G}, \tag{1g}$$

$$Q_{R_i}^{min} \leq Q_{R_i}^c \leq Q_{R_i}^{max}, \quad \forall i \in \mathcal{S}, \tag{1h}$$

$$\max |Q_{R_i}^c| \leq RPR_i, \quad \forall i \in \mathcal{S}\,, \tag{1i}$$

$$\delta_{slack} = 0, \tag{1j}$$

where $\mathcal{G}$, $\mathcal{N}$, and $\mathcal{T}$ are the sets of generators, buses, and lines. The set $\mathcal{N}_i$ contains the buses connected to bus i.

The set $\mathcal{S} \subseteq \mathcal{N}$ defines the candidate buses for additional reactive power supply allocation. The superscript c denotes the contingency index, with $c = 0$ for the base case. The decision variables V and δ are the voltage magnitudes and phase angles, P_G and Q_G are the active and reactive power outputs of existing generators, and Q_R and RPR are the reactive power needed during contingencies and the RPR requirement. The variable I_{ij} is the magnitude of the current flow on line (i, j). The notation $| \cdot |$ denotes the absolute value operator. The objective function (1a) consists of two terms: the summation of the active power generation cost and the summation of the absolute values (i.e., the L1 norm) of the RPR capacity costs at each candidate location. Constraints (1b) and (1c) are the AC power flow equations. Constraints (1d) and (1e) limit the line current flows and voltage magnitudes, while the generators' active and reactive power limits are enforced by (1f) and (1g). We use (1h) to limit the maximum capacity of the additional reactive power supply, and (1i) to determine the maximum reactive power supplied at each bus in $\mathcal{S}$ over all contingencies. The limits on the additional reactive power supply, $Q_{R_i}^{min}$ and $Q_{R_i}^{max}$, are user-defined parameters selected based on the technological capabilities of the reactive power supplies considered for addition. The reference angle is defined in (1j).

Explicitly enforcing a large number of contingencies leads to intractability of the SCOPF problem. Another challenge facing the posed RPR problem is that the buses considered for installing reactive support ($\mathcal{S}$) need to be determined before solving the problem. One approach for overcoming this challenge is to simply consider all buses as prospective candidates. However, the solution may allocate small quantities of reactive support scattered across the buses, which is not a practical solution. To overcome the two aforementioned challenges (tractability and bus selection), we screen the contingencies and pre-define candidate buses for reactive support installment, as discussed in the next section.

B. Selection Criteria

The solution of the SCOPF problem (1a)–(1j) provides a feasible operating point but does not directly provide information regarding voltage stability. We use the CPF method to identify the initial set of contingencies that will be considered in the SCOPF problem. The CPF method solves the power flow equations with small steps of demand increments until reaching a steady-state voltage stability limit (i.e., the critical point) [4]. By modifying the power flow equations and introducing a parameterization variable for the power injections, the CPF process avoids numerical difficulties associated with the singularity of the Jacobian matrix near the critical point. This allows calculation of the maximum achievable power transfer prior to the critical point along the continuation trajectory [15].

CPF is a predictor-corrector-based method. In the predictor step, the power trajectory is increased toward the tangent direction of the variable state, while the corrector step solves the power flow equations with the addition of a load parameterization equation corresponding to the loading trajectory. Using appropriate parameterization prevents the trajectory from passing the voltage stability limits; for more details, see [4]. The continuation trajectory is parameterized using the loading parameter λ, where $\lambda = 0$ under the base case loading and $\lambda = 1$ under the target loading case, i.e., the future loading scenario. We use a natural parameterization as defined in [16], where the value of the demand at the next step is equal to the previous step plus a constant step size. For the future scenarios considered in our analyses, the critical point is often reached before the target case, indicating infeasibility of the power flow solution at the target loading for this loading trajectory. The target loading is the future loading scenario obtained from a long-term forecast, e.g., a 10- to 20-year projection of expected demands. In our method, we select the contingency that reaches infeasibility with the lowest loading as the first candidate contingency in the SCOPF.

Adapting ideas from [4], we also use the output of the CPF to identify the "weakest bus" based on the bus sensitivities near the critical point. The definition of the weakest bus in this context is the bus with steepest slope near the critical point of the CPF curve, i.e., the bus with highest sensitivity in voltage with respect to demand. The weakest bus sensitivity is defined as:

$$\max_{i \in \mathcal{N}} \left| \frac{1}{C} \frac{dV_i}{d\lambda} \right| \quad \text{where} \quad C = \sum_{i \in \mathcal{N}} K_i \cos\theta_i, \tag{2}$$

V_i denotes the voltage magnitude, K_i is the step change in the demand, and $\cos(\theta_i)$ is the power factor of bus i near the critical point. Note that C is constant for each loading scenario. We use the value of dV_i, the voltage magnitude change near the critical point, to calculate the relative bus sensitivities within a particular scenario. We approximate the bus sensitivity index using the values obtained from the CPF at the critical point and the previous step.

C. Overall Methodology for Solving the RPR Problem

In this section, we combine the SCOPF formulation from Section II-A with the selection criteria in Section II-B in our proposed methodology for solving the reactive power allocation problem. The overall flow of the proposed methodology is shown in Fig. 2. We start by defining the scenarios, which include the base case, the future loading scenario, and the contingencies. We next apply the CPF to all contingencies considering the base case and the future loading scenario as the target. The results of the CPF are then used to select the contingency with the lowest loading at the critical point. For the selected case, we calculate the bus sensitivities and rank the buses from the weakest (largest sensitivity) to the strongest (lowest sensitivity) using (2). After that, we solve the SCOPF in (1a)–(1j) with the selected contingency and a predefined number of potential RPR locations, i.e., the weakest N_w buses.

If the solution is not feasible, we consider this infeasibility as an indication that the system has an inadequate supply of reactive power and hence further actions are needed. In this case, we increase the number of RPR candidate location to

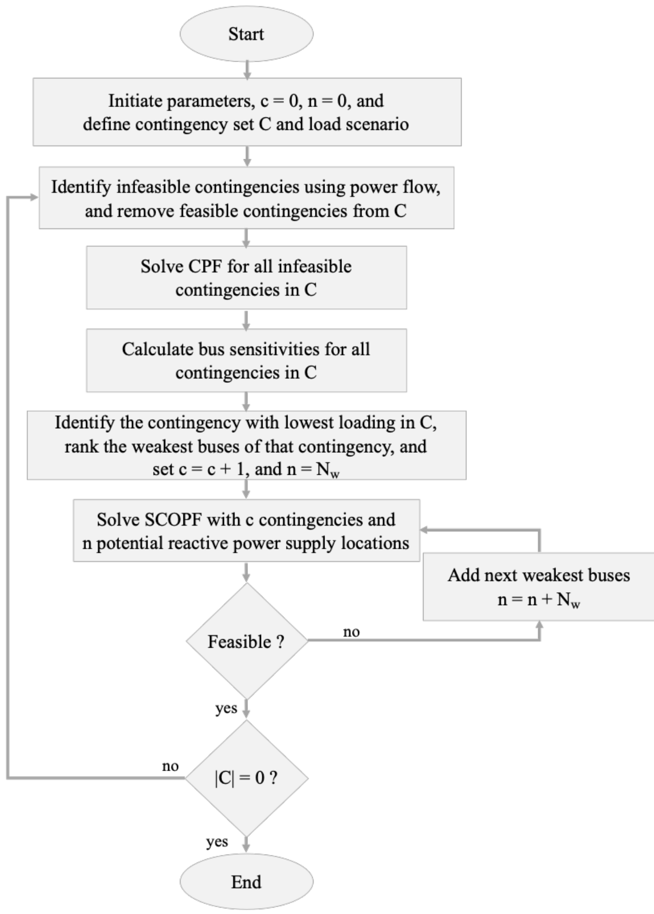

Fig. 2. Overview of the proposed RPR solution approach.

the next set of weakest buses. We repeat this process until we find a feasible solution for the selected contingency.

Once we obtain a feasible solution, the output of that iteration is taken as the solution with the lowest total reactive power required to meet the demand. We then check the operational constraints for all remaining contingencies by solving the power flow equations and eliminating contingencies where the operational constraints are satisfied.

We repeat the process of solving the CPF, calculating the sensitivities, and solving the SCOPF problem until the operational constraints are satisfied for all remaining contingencies. Note that after obtaining a feasible solution from the SCOPF problem and retrieving the optimal reactive power support installments, we keep the value of the reactive power constant in the consecutive iterations when solving the power flow equations to identify any violations of the operational constraints for the remaining contingencies.

III. Numerical Results

To validate the proposed method, we use the IEEE 30- and 118-bus systems [17]. We use the CPF method implemented in MATPOWER and the MATPOWER Interior Point Solver (MIPS) to solve the SCOPF [15].

For the 30-bus system, we consider a future loading scenario where the base case loading is multiplied by a factor of 2.5 with a constant power factor. We increase the generators' active power limits by the same factor while keeping the reactive power limits constant. This scenario represents an increase in renewable generation operated at unity power factor. The total demand in the base case is 189.2 MW, while the total demand in the target case is 473 MW. The total inductive demand in the base case is 107.2 MVAr, while the target case has a total demand of 268 MVAr. We consider the N-1 security criterion for all line contingencies except for the lines connected radially to a single bus.

Two contingencies were found to be infeasible for the future scenario: failures of the lines connecting buses 1 to 2 and 6 to 8. The bus sensitivities for the first contingency are shown in Fig. 2. We observe that the normalized bus sensitivities vary between 0.8 to 1 for all buses. The algorithm selected bus 8 with 70.2 MVAr as the location for additional reactive power supply. The CPF results before and after placing the reactive power supply for the most severe contingencies are shown in Fig. 4. In contingency 1 (line 1-2), we notice the critical point occurs when the loading parameter reaches 0.5 as indicated by the dashed lines in Fig. 4, while the loading parameter reaches 0.6 in contingency 2 (line 6-8). These values indicate that the system is not able to supply more than 50% and 60% of the target loading after contingencies 1 and 2, respectively. However, after installing the reactive support obtained from the proposed method, the system can reach the target loading under both contingencies as shown by the fact that the loading parameter exceeds 1.0.

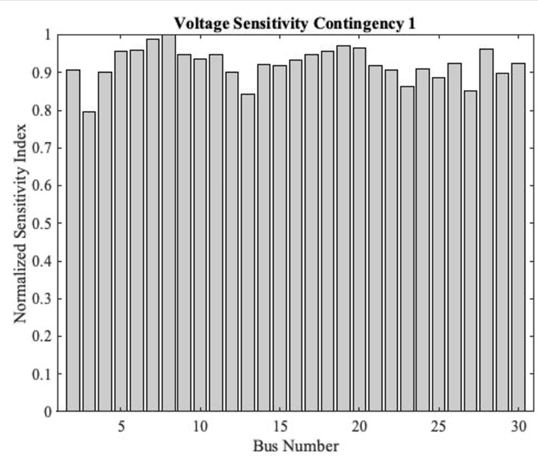

Fig. 3. Bus sensitivities for the IEEE 30-bus case.

For the IEEE 118-bus system, we consider a future loading scenario with increased loading by a factor of 2.5 from the base case for both active and reactive power demand. The total active power demand for the base case is 4242 MW, while the target case has a total active power demand of 10605 MW. The total inductive demand in the base case is 1438 MVAr, while the total inductive demand in the target case is 3595 MVAr. We consider two scenarios for this case. In the first scenario, we

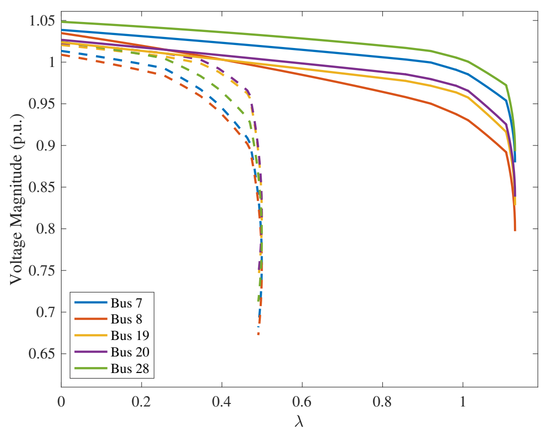

Fig. 4. CPF results before (dashed lines) and after (solid lines) applying the proposed method for the IEEE 30-bus case.

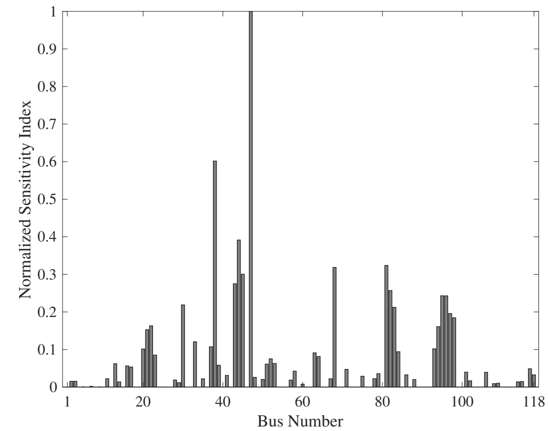

Fig. 5. Bus sensitivities after a contingency on the line between buses 69 and 75 in the first scenario for the IEEE 118-bus case.

increase the active power supply with the same loading factor while keeping the reactive power supply limits fixed similar to the previous case, while in the second scenario we remove 50% of the existing reactive power supply. For both scenarios, we consider a total of 177 contingencies corresponding to all N-1 line contingencies expect for the lines connected radially to a single bus.

Considering the first scenario for the IEEE-118 bus system without removing the existing reactive power supplies, we identify 28 contingencies that are infeasible at the target loading. The value of the loading parameter λ at the critical point for those contingencies varies between 0.53 to 0.59. Furthermore, we observe that bus 47 has the highest sensitivity index during all the identified contingencies. As a representative example, the bus sensitivities for one of the contingencies with the lowest loading parameters λ are shown in Fig. 5. The proposed algorithm augments bus 45 with an additional reactive power supply of 13.65 MVAr. With this added reactive power supply, the future loading scenario has a feasible operating point. Fig. 6 shows the CPF results before and after adding the additional reactive power supply for the contingency of losing the line connecting bus 69 and bus 75.

For the second scenario where we reduce the reactive power supply by 50%, the base case with target loading scenario becomes infeasible due to the lack of reactive power supply. Using the CPF method, we found that the maximum loading parameters λ for the base case without considering any line contingency is 0.59. During the line contingencies, the value of the maximum loading parameter λ is as low as 0.19. The contingencies with the lowest loading parameters λ are shown in Table I.

The bus sensitivities for the lowest loading parameter contingency, the loss of the line between bus 65 and bus 68, are shown in Fig. 7. The algorithm selects five locations for the installation of the addition reactive power supply as shown in Table II. Further, as shown in Fig. 8, the CPF results demonstrate that the total loading of the system can exceed

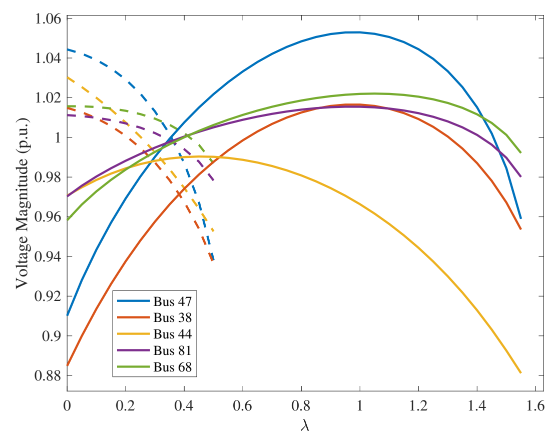

Fig. 6. CPF results for the weakest busses before (dashed lines) and after (solid lines) applying the proposed method to the first scenario for the IEEE 118-bus case.

the target loading scenario at the the contingency with lowest loading parameter.

IV. Conclusion

Ensuring adequate RPR planning is crucial to securely operate the power system. Providing fast reactive power during contingencies prevents the system from exceeding the VSM. However, we need to be selective when allocating the RPR due to the high cost of the reactive power supply. We propose an algorithm based on the SCOPF problem in this paper to solve the reactive power reserves siting and sizing problem. The typical SCOPF can be intractable when considering a large number of contingencies. In addition, selecting prospective buses for installing additional reactive support presents another challenge. This paper has proposed a methodology for solving the RPR allocation problem which combines several heuristic techniques based on bus sensitivities and CPF calculations. The proposed methodology systematically allocates RPR supplies while considering a large set of contingencies.

TABLE I
LINE CONTINGENCIES WITH LOWEST LOADING PARAMETER $lambda$.

Cong. #	From Bus	To Bus	Maximum λ
1	65	68	0.19
2	68	69	0.24
3	38	65	0.42
4	68	81	0.45
5	81	80	0.45
6	23	24	0.46
7	69	70	0.49
8	69	77	0.51
9	69	75	0.53
10	70	71	0.54

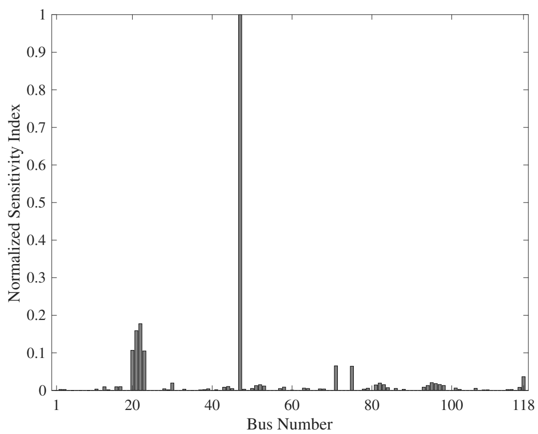

Fig. 7. Bus sensitivities of line contingency between Bus 65 and Bus 68 for the second scenario for the IEEE 118-Bus case.

Furthermore, bus sensitivities are used to select the potential candidate for reactive power supply to reduce the number of the additional RPR locations. For future work, we aim to use global optimization and convex relaxation technique to assess the quality of the solution from the proposed methodology and compare the results with alternative heuristics.

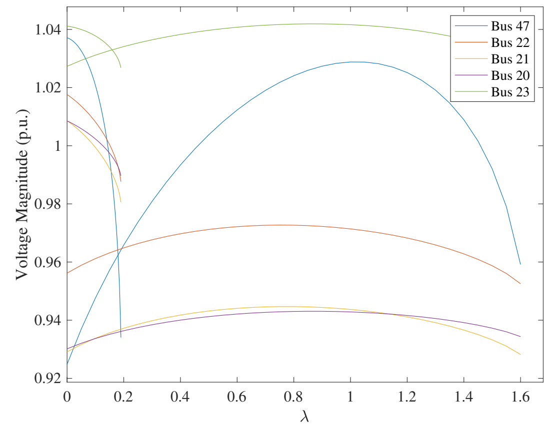

Fig. 8. CPF results for the weakest busses before (dashed lines) and after (solid lines) applying the proposed method to the second scenario for the IEEE 118-bus case.

TABLE II
SELECTED RPR LOCATIONS AND CAPACITIES.

Location #	Bus	Capacity (MVAr)
1	82	19.29
2	94	66.97
3	95	55.12
4	98	9.24
5	118	70.24

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