Sample-Based Conservative Bias Linear Power Flow Approximations

Paprapee Buason, Sidhant Misra, Daniel K. Molzahn

***Abstract*—The power flow equations are central to many problems in power system planning, analysis, and control. However, their inherent non-linearity and non-convexity present substantial challenges during problem-solving processes, especially for optimization problems. Accordingly, linear approximations are commonly employed to streamline computations, although this can often entail compromises in accuracy and feasibility. This paper proposes an approach termed *Conservative Bias Linear Approximations (CBLA)* for addressing these limitations. By minimizing approximation errors across a specified operating range while incorporating conservativeness (over- or under-estimating quantities of interest), CBLA strikes a balance between accuracy and tractability by maintaining linear constraints. By allowing users to design loss functions tailored to the specific approximated function, the bias approximation approach significantly enhances approximation accuracy. We illustrate the effectiveness of our proposed approach through several test cases.**

***Index Terms*—Conservative bias linear approximation; power flow approximation**

I. Introduction

The power flow equations play a central role in the operation and analysis of electrical power systems. These equations are essential for evaluating the behavior of power networks, making them key to various optimization problems such as resilient infrastructure planning [1]–[3], AC unit commitment [4], [5], and bilevel problems [6], [7]. However, the nonlinearity of the power flow equations induces non-convexities in these problems that pose significant computational challenges.

To address these challenges, researchers have developed various linear approximations such as DC power flow [8], LinDistFlow [9], first-order Taylor expansions of the power flow equations, and other approximations [10]. These methods offer simplified representations of power flow, which improve the tractability of power systems optimization problems. However, these linearizations often depend on broad assumptions such as maintaining voltages at 1 per unit and keeping voltage angle differences small between neighboring buses, as in DC power flow. These assumptions may not be valid across all operating conditions, potentially resulting in inaccuracies in the approximations. Consequently, the solutions derived from these linearized models may not closely align with the actual optimal solutions in real-world scenarios. This trade-off between simplicity and accuracy necessitates careful consideration when applying these linearizations in practice.

In response to these challenges, various studies have explored *adaptive* power flow approximations tailored to specific systems and operating ranges to enhance approximation accuracy (e.g., optimization-based approaches in [11] and sample-based approaches in [12]–[14]); see [15]–[17] for recent survey papers on this concept. For sample-based approaches, samples of operating points computed by repeatedly solving the power flow equations at various points within an operating range (e.g., a specified range of power injections) are leveraged to compute the approximations. By capturing complex nonlinear relationships directly from these samples, the resulting linear approximations can be more accurate as they are formulated by minimizing deviations from the solutions provided by the AC power flow equations for a particular system and operating range of interest. These adaptive power flow linearizations spend computational effort in computing the linearization coefficients in order to improve accuracy and tractability when applied in optimization problems. By trading up-front computational time for increased accuracy when applied, adaptive power flow approximations are particularly valuable in settings with both offline and online aspects where the linearization coefficients can be computed offline in advance of a real-time problem as well as settings where explicitly modeling power flow nonlinearities would lead to intractability, e.g., [1]–[7].

Extending the concept of sample-based adaptive power flow linearizations, the conservative linear approximation (CLA) approach in [12], [13] incorporates the concept of *conservativeness*. In other words, the CLAs are computed to minimize approximation errors with respect to the AC power flow equations while consistently over- or under-estimating quantities of interest over the set of drawn samples. The resulting approximations are particularly well suited for settings with an asymmetry in the implications of overestimating a quantity like voltage magnitude or current flow as opposed to underestimating that quantity. This is particularly relevant in power system optimization problems where feasibility is of paramount importance. For instance, when used in the bound on the magnitude of current flow through a line, a linearization that erroneously underestimates the amount of current flow risks predicting feasibility when the constraint is actually violated with respect to the nonlinear AC power flow equations. This is a more problematic linearization error than an overestimate of the current flow for use in this constraint. Thus, conservative linearizations that avoid errors in a particular direction (i.e., avoid either overestimates or underestimates of some quantity) are valuable in many power system optimization contexts.

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However, maintaining conservativeness can sometimes lead to reduced accuracy.

In this paper, we introduce an approach to approximating power flow equations called *conservative bias linear approximation* (CBLA). The CBLA approach seeks to balance the trade-off between conservativeness and accuracy, particularly in scenarios where certain samples are challenging to approximate accurately. To construct CBLAs, the process shares similarities with CLAs by beginning with drawing samples from within the operational range. These samples form the basis of a regression problem, which is solved to compute an approximated function representing the power flow equations. However, unlike CLA, CBLA does not explicitly enforce conservativeness in its approximated function as a hard constraint. Instead, the CBLA approach introduces an error function that penalizes linearization errors for samples that violate conservativeness to enable more accurate approximations.

CBLA offers the advantage of flexibility in designing customized error functions that quantify the penalty for deviating from actual values. User-defined error functions enable the approach to be tailored to particular quantities of interest and system characteristics, thus computing a linearization specialized for a specific problem. This flexibility can be particularly beneficial in scenarios where some violations are permissible, such as in chance-constrained optimization problems.

In summary, the main contributions of this paper are:

(*i*) A CBLA formulation that is tailored to the specific system and operating range, optimal with respect to an error metric, and strikes a balance between conservativeness and accuracy.
(*ii*) A discussion on choosing an error function for computing the CBLA.
(*iii*) Numerical benchmarking of CBLAs for a variety of test cases.

The remainder of this paper is organized as follows: Section II covers background material on the power flow equations as well as sample-based conservative linear approximations. Section III introduces the conservative bias linear approximation approach. Section IV provides numerical results of our approach. Section V concludes the paper along with directions for our future work.

II. Background

In this section, we provide background information about the AC power flow equations and present the recently developed conservative linear approximations of these equations.

A. The Power Flow Equations

Consider a power system where a reference bus has the voltage angle set to 0. Let $V(\theta)$ denote the voltage magnitude (phasor). Let $P(Q)$ denote the active (reactive) power injection. We use the subscript $(\cdot)_i$ to represent a quantity at bus i and the subscript $(\cdot)_{ik}$ to represent a quantity from or connecting bus i to k. Let $j = \sqrt{-1}$. The AC power flow equations at bus i are:

$$P_i = V_i^2 G_{ii} + \sum_{k \in \mathcal{B}_i} V_i V_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}), \quad (1a)$$

$$Q_i = -V_i^2 B_{ii} + \sum_{k \in \mathcal{B}_i} V_i V_k (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}), \quad (1b)$$

where $\theta_{ik} := \theta_i - \theta_k$, $G(B)$ is a real (imaginary) part of the admittance matrix associated with the system, and $\mathcal{B}_i$ represents the set of all neighboring buses to bus i including bus i itself.

B. Conservative linear approximations

The nonlinearity of the power flow equations in (1) contribute to the complexity encountered in solving optimization problems. To address this challenge, we previously introduced a sample-based conservative linear approximation (CLA) approach aimed at either over- or under-estimating specified quantities of interest, such as the magnitudes of voltages and current flows (as illustrated in Fig. 1) [12]. Moreover, CLAs facilitate parallel computation by enabling concurrent computation of the CLA for each quantity of interest. The construction of a CLA entails sampling power injections across an operational range of interest, such as a range of loads and power generated by Distributed Energy Resources (DERs), followed by computing power flow solutions for each sample and solving a constrained-regression problem.

For instance, samples for load demands are acquired utilizing a predefined probability distribution $\mathbb{P}_\mathcal{S}$ over a specified operational range $\mathcal{S}$. This range could be defined as $\mathcal{S} = \{P_{L_d}^{\min} \leq P_{L_d} \leq P_{L_d}^{\max},\ Q_{L_d}^{\min} \leq Q_{L_d} \leq Q_{L_d}^{\max}$ for all $d \in \mathcal{N_D}\}$ where $(\cdot)_{L_d}$, where $(\cdot)_{L_d}$ denotes the load demand, $\mathbb{P}_\mathcal{S}$ represents the uniform distribution, and the superscripts max (min) indicate upper (lower) limits.

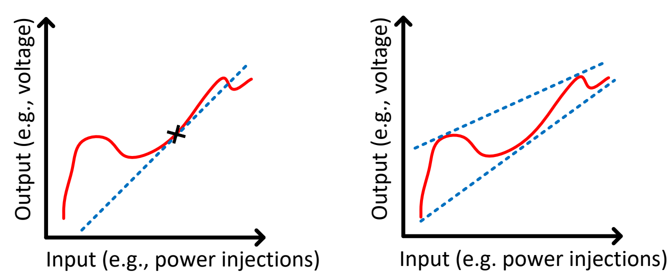


Fig. 1. An illustration showcasing a comparison between a conventional linear approximation (on the left) and CLAs (on the right). In this visual representation, the solid line signifies the nonlinear function under consideration. In the figure on the left, the dotted line represents a traditional first-order Taylor approximation centered at point $\times$, while in the figure on the right, the dotted line above (below) corresponds to an over- (under-)estimating approximation.

The utilization of CLAs allows for the customization of the approximation to fit a specific operating range and the targeted system. Additionally, the sample-based approach enables integrating the behavior of complicated devices like tap-changing transformers and smart inverters into the approximation, as discussed in our prior work [6]. In the realm of optimization, CLAs offer a crucial advantage: they enable the satisfaction of nonlinear constraints while enforcing only linear inequalities, assuming the CLAs maintain conservativeness. Consequently, CLAs streamline optimization problems, rendering them suitable for commercial optimization solvers.

Consider a quantity of interest denoted as γ, which could represent variables such as the voltage magnitude at a specific bus or the magnitude of current flow along a particular line. In this context, bold quantities signify matrices and vectors. Let superscript T denote the transpose. An *overestimating* CLA can be expressed as follows:

$$a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix} \tag{2}$$

where a_0 is a scalar and $\boldsymbol{a}_1$ is a vector, both serving as decision variables in the regression problem later described in (4). This CLA is constructed to ensure the fulfillment of the following relationship for power injections $\boldsymbol{P}$ and $\boldsymbol{Q}$ within a specified range:

$$\gamma \leq a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix}. \tag{3}$$

Assuming that (3) is indeed satisfied, we can ensure that the constraint $\gamma \leq \gamma^{\max}$ is also satisfied by instead enforcing a linear constraint $a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix} \leq \gamma^{\max}$. This approach allows us to meet the upper bound requirement $\gamma^{\max}$ without introducing the implicit system of nonlinear AC power flow equations in (1). Importantly, by employing the CLA, we are able to satisfy the nonlinear equations while maintaining a linear formulation, thus enhancing computational tractability without sacrificing feasibility in the resulting solution.

To compute a CLA, we solve for the coefficients of the affine function of power injections in (2) in the following regression problem:

$$\min_{a_0,\ \boldsymbol{a}_1} \quad \frac{1}{M}\sum_{m=1}^{M} \mathcal{L}\left(\gamma_m - \left(a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P}_m \\ \boldsymbol{Q}_m \end{bmatrix}\right)\right) \tag{4a}$$

$$\text{s.t.} \quad \gamma_m - \left(a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P}_m \\ \boldsymbol{Q}_m \end{bmatrix}\right) \leq 0, \quad m = 1, \ldots, M. \tag{4b}$$

The subscript $(\,\cdot\,)_m$ denotes the m^{th} sample and M is the number of samples. The function $\mathcal{L}(\,\cdot\,)$ represents a loss function, such as the absolute value for ℓ_1 loss or the square for squared-ℓ_2 loss. In this paper, our focus is on quantities of interest denoted by γ, which correspond to the magnitudes of voltages (V) and current flows (I). The construction of underestimating CLAs follows a similar process as described in (4), with the key distinction being the reversal of the inequality direction in (4b).

The conservativeness of the CLA computed in (4) comes at the cost of reduced accuracy relative to the approximation corresponding to the unconstrained regression problem resulting from dropping (4a) from (4). To manage this tradeoff, the next section presents the main contribution of this paper, namely, a linear approximation technique that achieves a balance between conservativeness and accuracy. This approach involves biasing the linearization towards conservativeness, guided by a designated loss function.

III. Conservative Bias Linear Approximations

The CLA approach presented in Section II-B is consistently conservative within the set of drawn samples. However, in certain scenarios, the conservativeness property may lead to significant errors due to specific samples. In this paper, we present a sample-based *conservative bias linear approximation* (CBLA) approach that is adaptive, meaning it can be tailored to a specific system and operating range. The CBLA approach is designed to be optimal, aiming to minimize a specific error metric while retaining a *tendency to be conservative* in order to enhance accuracy. This implies that the CBLA primarily minimizes errors between the approximating function and the samples, permitting samples to violate conservativeness at a specified cost.

A. Formulation

Let ϵ denote the mismatch between the approximated quantity and the actual quantity. The optimization problem to compute a CBLA is formulated as follows:

$$\min_{f(\epsilon_m(a_0, \boldsymbol{a}_1))} \quad \frac{1}{M}\sum_{m=1}^{M} f(\epsilon_m) \tag{5}$$

where

$$[\forall m = 1, \ldots, M]$$
$$\epsilon_m = \gamma_m - \left(a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P}_m \\ \boldsymbol{Q}_m \end{bmatrix}\right), \tag{6}$$

and

$$f(\epsilon_m) = \begin{cases} g(\epsilon_m), & \text{if } \epsilon_m \leq 0 \\ h(\epsilon_m), & \text{otherwise.} \end{cases} \tag{7}$$

The optimization problem in (5) seeks to minimize the aggregated value of the error function $f(\,\cdot\,)$ defined in (7) over all samples by computing the coefficients a_0 and $\boldsymbol{a_1}$ in (6). This error function is contingent upon the error mismatch, denoted as ϵ in (6), between the estimated quantity and the actual quantity (γ). The error function is computed based on the sign of the error for each sample. In cases of *overestimation*, the value of $h(\epsilon_m)$ is designed to be high, imposing a substantial cost for violation. Conversely, the value of $g(\epsilon_m)$ is intended to be relatively low, reflecting scenarios where samples do not violate the conservativeness. Vice-versa, the value of $h(\epsilon_m)$ is designed to be relatively low and $g(\epsilon_m)$ to be high for an *underestimation*.

B. Error function

Choosing a suitable error function in the CBLA approach is an important consideration. The choice of error function depends on various factors, such as the specific system requirements, a quantity of interest, and the trade-off between accuracy and conservativeness. To better understand how the error function works, we compare the error function used in the CLA approach with that of the CBLA approach.

The CLA approach imposes conservativeness across the set of sampled data in the constraints. We can rewrite the regression problem described in (4), which utilizes the ℓ_1 loss function, as an optimization problem formulated in (5)–(7). In this formulation, the error function is defined as follows:

$$f(\epsilon_m) = \begin{cases} \epsilon_m, & \text{if } \epsilon_m \leq 0 \\ \infty, & \text{otherwise.} \end{cases} \tag{8}$$

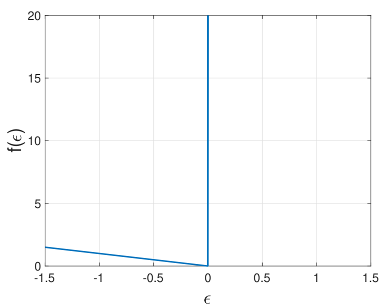


Fig. 2. An error function of the CLA where $g(\epsilon) = \epsilon$ and $h(\epsilon) = \infty$.

The error function defined in equation (8) (see Fig. 2) assigns an infinite cost to any violation of the overestimating requirement. This implies that for all samples drawn, no violation is permitted.

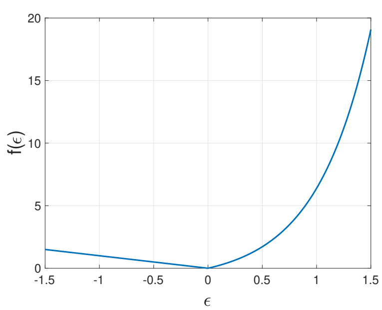


Fig. 3. An example of an error function where $g(\epsilon) = \epsilon$ and $h(\epsilon) = e^{2\epsilon} - 1$.

In contrast, our CBLA approach offers the flexibility to configure an error function that accommodates violations for specific samples, all while considering a predefined cost associated with these violations. This ability to tailor the error function empowers us to strike a balance between accuracy and the acceptable level of conservativeness for the specific system and operating range of interest. In Fig. 3, an example of an error function $f(\epsilon)$ for an overestimating CBLA is depicted. The function $f(\epsilon)$ exhibits a high value for $\epsilon > 0$, $f(\epsilon)$ maintains a relatively low value for $\epsilon < 0$, and $f(0)$ is zero, indicating an exact approximation.

In this setup, by assigning a higher penalty when an approximation violates conservativeness, we incline towards more conservative approximations while sacrificing some accuracy. Additionally, when the derivative of the function $h(\epsilon)$ increases as shown in Fig. 3, this function likely permits only small positive values of ϵ, as larger positive values result in exponentially higher penalties. Conversely, reducing the cost tends to yield more accurate approximations at the expense of conservativeness.

When the error function is defined as a piecewise linear function (i.e., when both $g(\epsilon)$ and $h(\epsilon)$ are linear functions), the problem formulation to compute CBLAs in (5)–(7) can be framed as a linear program as follows:

$$\min_{a_0,\ \boldsymbol{a}_1} \quad \frac{1}{M}\sum_{m=1}^{M} z_m \tag{9a}$$

$$\text{s.t.} \quad [\forall m = 1, \ldots, M]$$

$$\epsilon_m = \gamma_m - \left(a_0 + \boldsymbol{a}_1^T \begin{bmatrix} \boldsymbol{P}_m \\ \boldsymbol{Q}_m \end{bmatrix} \right), \tag{9b}$$

$$z_m \geq k_1 \epsilon_m, \tag{9c}$$

$$z_m \geq k_2 \epsilon_m, \tag{9d}$$

where z is a slack variable, and k_1 and k_2 are the coefficients of the linear error functions, i.e., $g(\epsilon) = k_1\epsilon$ and $h(\epsilon) = k_2\epsilon$.

With nonlinear error functions, the regression problem is a mixed-integer nonlinear program; nevertheless, it can be conveniently implemented using a user-defined function in Julia and subsequently solved with a Julia package like Optim, which provides a framework for solving constrained optimization problems [18].

IV. Numerical Results

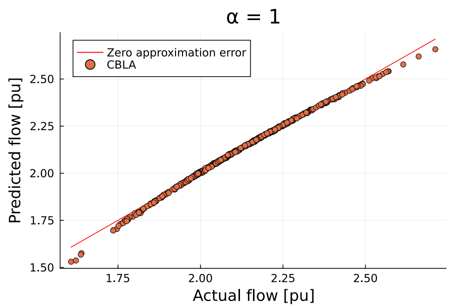
In this section, we conduct numerical experiments on several test cases to examine the behavior of CBLA, highlight the benefits of error function design, and demonstrate the effectiveness of CBLA in a simplified OPF problem.

The test cases used in the simulations are *case6ww*, *case14*, and the IEEE 24-bus system, all of which are accessible in MATPOWER [19]. For approximations of voltage and current flow, we draw 500 samples by varying the power injections within a range of 70% to 130% of their nominal values. Both voltage and current flow values are reported in per unit (pu). We use the ℓ_1 norm as the loss function $\mathcal{L}(\,\cdot\,)$. The numerical simulation was conducted in Julia using the Optim package.

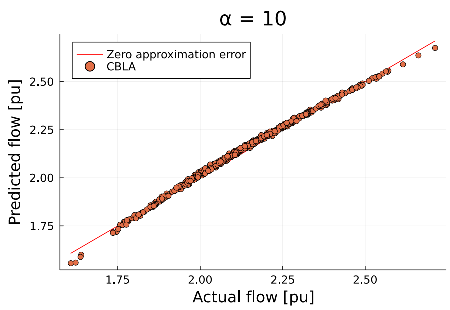
A. Conservative Bias Linear Approximations

We begin our numerical tests by examining the effects of changing the error function in (7) in our CBLA approach. As discussed in Section III-B, error functions are designed to balance conservativeness and accuracy. By testing different error functions, we aim to understand how they impact the number of violated samples and accuracy of the approximated power flow equations.

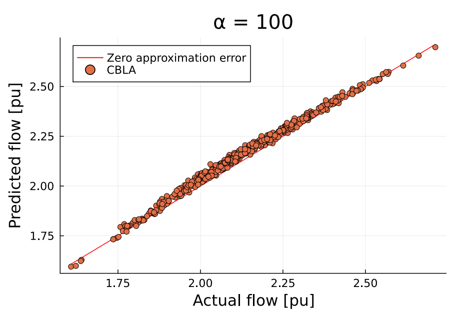
In Fig. 4, we present the results of using the CBLA approach to intentionally overestimate the predicted current flow from bus 3 to bus 24 in IEEE 24-bus system using different quadratic error functions. The error functions employed in this test are defined as $g(\epsilon) = x^2$ and $h(\epsilon) = \alpha x^2$, where α is a parameter that we vary across the test. Specifically, we adjust α to take values of 1, 10, 100, and 1000. When $\alpha = 1$, the error functions are equivalent (i.e., $g(\epsilon) = h(\epsilon)$), indicating that there is no difference in cost between violating and not violating conservativeness (i.e., the error function is the squared-ℓ_2 loss). Under this condition, almost all samples



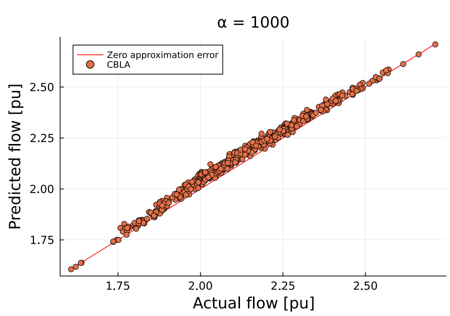
(a) $g(\epsilon) = \epsilon^2, h(\epsilon) = \epsilon^2$.



(b) $g(\epsilon) = \epsilon^2, h(\epsilon) = 10\epsilon^2$.



(c) $g(\epsilon) = \epsilon^2, h(\epsilon) = 100\epsilon^2$.



(d) $g(\epsilon) = \epsilon^2, h(\epsilon) = 1000\epsilon^2$.

Fig. 4. Plots of results when (a) $\alpha = 1$ (equivalent to the squared-ℓ_2 loss), (b) $\alpha = 10$, (c) $\alpha = 100$, and (d) $\alpha = 1000$ for current flow from bus 3 to bus 24 in IEEE 24-bus system. The red points represent overestimating CBLAs. The black line represents a zero approximation error.

are well approximated, but several samples fall below the zero approximation error line, indicating a deviation from the overestimating objective.

As we increase α, fewer samples fall below the zero approximation error line, suggesting improved adherence to the overestimation goal. However, this improvement comes at the cost of lower overall accuracy. At the highest value of $\alpha = 1000$, most samples are overestimated as intended, but the level of accuracy appears to be the lowest among all the plots. This trade-off highlights the importance of carefully selecting the value of α to achieve a suitable balance between overestimation and accuracy.

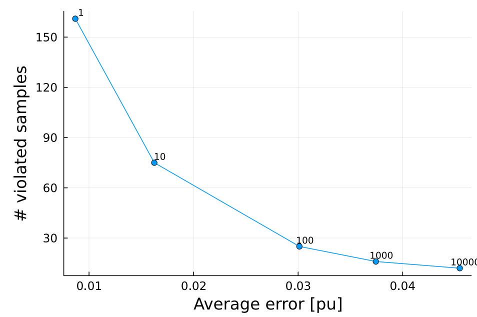


Fig. 5. Results showing the average error per sample in per unit (pu) and the number of violated samples due to overestimating CBLA of current flow from bus 3 to bus 24 in IEEE 24-bus system, as the value of α (labeled at each point) varies from 1 to 10^4.

To gain further insight into the effects of varying α, we plot the relationship between the average error per sample of the approximated flow and the number of violated samples when varying the value of α in Fig. 5. In this test, we adjust α over a range from 1 to 10^4. The results reveal a clear trend: as α increases, the average error per sample also increases while the number of violated samples decreases significantly, demonstrating the trade-off between conservativeness and accuracy. This is due to the increased enforcement of conservativeness in the error function. Specifically, the average error per sample increases from 0.00869 when $\alpha = 1$ to 0.0455 when $\alpha = 10^4$, while the number of violated samples decreases from 161 when $\alpha = 1$ to just 12 when $\alpha = 10^4$.

TABLE I
APPROXIMATED CURRENT FLOW ERRORS AND NUMBER OF VIOLATED SAMPLES AT SPECIFIC BUSES IN IEEE 24-BUS SYSTEM

Line	Average errors/sample			# violated samples		
(From-to)	$\alpha = 1$	$\alpha = 10^2$	$\alpha = 10^4$	$\alpha = 1$	$\alpha = 10^2$	$\alpha = 10^4$
3-14	0.00869	0.03012	0.04551	161	25	12
6-10	0.00907	0.02274	0.03780	202	30	8
9-12	0.01621	0.04961	0.09397	180	29	6

The data presented in Table I illustrates the relationship between the number of violated samples and the average approximated current flow errors across different values of α at different lines. These results align with the trend observed in Fig. 5, confirming that as α increases, the average error per sample increases while the number of violated samples decreases.

B. Application: Simplified optimal power flow

While our main goal is to use our CBLA approach for challenge problems such as bilevel problems and mixed-integer nonlinear programs, these are outside the scope of this paper. Instead, we concentrate on showcasing results within a simplified optimal power flow (OPF) framework, which offers a conceptual demonstration and a foundation for comparing different linear approximations (DC power flow and conservative bias linear approximation (CBLA)). This

TABLE II
RESULTS FROM OPF COMPARING SOLUTIONS FROM AC-, DC-, AND CBLA-OPF

Formulation	Case	
	case6ww	*case14*
AC-OPF	2986.04	5368.30
DC-OPF	2995.15 (0.31%)	5368.52 (0.004%)
Violation	V (0.029 pu)	No violation
CBLA-OPF ($\alpha = 1$)	2987.28 (0.04%)	5368.52 (0.004%)
Violation	No violation	V (0.004 pu)
CBLA-OPF ($\alpha = 10^2$)	2987.42 (0.05%)	5368.52 (0.004%)
Violation	No violation	No violation
CBLA-OPF ($\alpha = 10^6$)	2987.51 (0.05%)	5368.52 (0.004%)
Violation	No violation	No violation

simplified version of the OPF scenario imposes constraints on voltages at buses where P and Q are known, and power generation within defined ranges, excluding line flow limits (see [12] for the full problem setup).

Table II presents the outcomes of applying the AC power flow equations and various power flow approximations in a simplified optimal power flow (OPF) scenario. The AC-OPF serves as the baseline, with no violations and providing a reference for comparing the percentage difference in optimal cost against other formulations. In the overestimating CBLA approach, we utilize error functions $g(\epsilon) = \epsilon^2$ and $h(\epsilon) = \alpha\epsilon^2$, while for underestimating CBLA, we use error functions $g(\epsilon) = \alpha\epsilon^2$ and $h(\epsilon) = \epsilon^2$. As discussed in Section IV-A, setting $\alpha = 1$ indicates that the error function does not differentiate between overestimating or underestimating the linearization error.

The results indicate that the DC-OPF leads to a maximum voltage violation of 0.029 pu and an optimal cost that is 0.31% higher than the AC-OPF's solution for *case6ww*. In *case14*, the CBLA-OPF causes only a minor voltage violation of 0.004 pu when $\alpha = 1$. When $\alpha = 10^2$ and 10^6 (with the latter closely representing the conservative linear approximations in our previous work [12]), both test cases have no voltage violations. Moreover, the optimal costs using DC-OPF and CBLA-OPF are only 0.004% higher than the optimal solution from AC-OPF in *case14*. These results suggest that the flexibility of CBLA allows us to select a suitable function that optimizes cost depending on the specific system being analyzed.

V. CONCLUSION AND FUTURE WORK

This paper presents a conservative bias linear approximation (CBLA) approach for approximating the power flow equations. This approach strives to balance conservativeness and accuracy while maintaining linearity in the approximations. The numerical results highlight the potential advantages of using CBLA for power flow problems. By selecting an appropriate error function, we can achieve an effective balance between conservativeness and accuracy. Additionally, the ability to choose different error functions allows CBLA to be tailored to specific systems and operational conditions, ultimately enhancing performance and reliability in power system optimization.

In our future work, we aim to extend our current approach by developing additional conservative bias approximations through the use of piecewise linearizations formulated as neural networks. Moreover, we plan to apply our proposed approach to a broader range of power system planning and resilience tasks. This includes tackling complex bilevel problems as well as conducting capacity expansion planning studies.

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