On The Relationships Among Different Voltage Unbalance Definitions

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***Abstract*—Growing penetrations of distributed energy resources (DERs) increase the power injection variability in distribution systems, which can result in power quality issues such as voltage unbalance. To measure unbalance, organizations such as IEC, NEMA and IEEE define phase unbalance in their power quality standards. However, the definitions in these different standards are not consistent, and voltages that are considered acceptable by one standard may violate good practices defined by another standard. To address this issue, this paper provides analytical comparisons of the most common voltage unbalance definitions, which are supplemented with numerical simulations. The analytical relationships suggest that it is possible to approximately bound the symmetrical-component-based voltage unbalance factor (which depends on the magnitude and relative phase angle) by limiting the line-to-line voltage unbalance, whereas applying line-to-ground voltage unbalance definitions neglects all information about phase angle offsets.**

I. Introduction

Increasing penetration of distributed energy resources (DERs) are resulting in greater variability of the net load in distribution systems. This challenges distribution system operators' ability to maintain acceptable power quality and may exacerbate problems such as voltage unbalance, which is the focus of this paper. Unbalanced operating conditions can damage power system equipment, such as three-phase induction motors [1], and increase network losses [2]. Distribution system operators strive to maintain balanced voltages by equally distributing load demands on all three phases of distribution feeders [3]. Variability associated with DERs can cause time-varying changes in the per-phase loading. Thus, high penetrations of DERs can challenge traditional approaches for balancing the net loading among the three phases in distribution systems.

Organizations such as IEC, NEMA and IEEE have proposed power quality standards regarding voltage unbalance. However, the associated definitions of voltage unbalance are not consistent. The IEC standard [4] uses a definition based on the positive and negative sequences from the symmetrical component transformation [5], which requires measurement of both voltage magnitudes and relative phase angles. Standards from NEMA [6] and IEEE [7] define phase unbalance using line-to-line and line-to-ground voltage magnitudes, respectively.

This work was funded by the U.S. Department of Energys Office of Energy Efficiency and Renewable Energy (EERE) under Solar Energy Technologies Office (SETO) Agreement Number 34235.

There are a number of papers analyzing the differences among the voltage unbalance definitions. In [8] and [9], three definitions of voltage unbalance (IEEE 1159, IEEE 141-1993, and NEMA) are numerically compared. While the overall conclusion of these papers is that voltage unbalance definitions agree reasonably well for unbalances below 5%, the numerical comparisons show that there are still differences. This implies that operating points which satisfy one power quality standard might violate other standards, which motivates a more thorough analysis of the relationships among various voltage unbalance definitions. Several approximations for the IEC definition based only on line-to-line voltage magnitude measurements (thus avoiding angle measurements) are analyzed in [10] and [11]. However, the resulting expressions are highly non-linear and no clear error bounds are provided.

Relative to existing literature, we do not just numerically evaluate the differences and similarities among voltage unbalance definitions (which is useful if we want to use one definition as an approximation for the others). Instead, our focus is on providing bounds on the *maximum* difference among the voltage unbalance definitions. This is helpful for understanding how to use one voltage unbalance definition (which may, e.g., be easier to measure in real time) to infer safe bounds on the voltage unbalance as defined by another definition. In particular, we are interested in how measurements of line-to-ground and line-to-line voltage magnitudes, which allow us to compute unbalance using IEEE and NEMA definitions, can be used to enforce limits on the voltage unbalance factor defined by the IEC standard. This is a practically important task, as access to measurements such as relative voltage angles are frequently unavailable. Furthermore, while previous papers have relied solely on numerical simulations, we derive bounds based on analytical expressions and then verify those bounds numerically.

In summary, the main contributions of this paper are: 1) derivations of analytical relationships among the voltage unbalance definitions, 2) empirical evaluations characterizing these relationships' accuracy, and 3) assessment of the conditions under which these relationships are most inaccurate.

The rest of the paper is structured as follows: Section II introduces various voltage unbalance definitions. Section III provides analytical analyses and numerical experiments that characterize voltage unbalances according to these definitions. Section IV concludes the paper.

II. VOLTAGE UNBALANCE DEFINITIONS

Voltage unbalance arises from asymmetric operation leading to phase voltages with unequal magnitudes, phase shifts that are not equal to 120 degrees, or a combination of both [5]. This section presents three common definitions for voltage unbalance from IEC [4], NEMA [6], and IEEE [7]. Other unbalance definition based solely on line-to-line voltage magnitudes are used by instrument manufacturers for power quality measurements as an approximation to the IEC definition [11]. These approximations are not discussed in this paper as they do not represent fundamentally different definitions, but are rather approximations of one of the considered definitions.

We use the following notation to represent phasors: $\mathbf{V} = V\angle\theta$, where $\mathbf{V}$ denotes the voltage phasor as a complex number, V is the voltage magnitude, and $\angle\theta$ is the phase angle. Denote the complex conjugate of $\mathbf{V}$ as $\overline{\mathbf{V}}$. Let $j = \sqrt{-1}$.

A. IEC definition (VUF)

Unbalanced voltages can be described using symmetrical components. The definition of the Voltage Unbalance Factor (VUF) frequently considered to be the "true" definition of voltage unbalance is based on the relative magnitudes of the negative and positive sequence voltages. The VUF definition adopted by the IEC [4] is

$$VUF\,[\%] = \frac{|\mathbf{V}_n|}{|\mathbf{V}_p|} \times 100, \tag{1}$$
$$\text{where}\ \ \mathbf{V}_p = \frac{\mathbf{V}_a + \mathbf{a}\cdot\mathbf{V}_b + \mathbf{a}^2\cdot\mathbf{V}_c}{3},$$
$$\mathbf{V}_n = \frac{\mathbf{V}_a + \mathbf{a}^2\cdot\mathbf{V}_b + \mathbf{a}\cdot\mathbf{V}_c}{3}.$$

In (1), $\mathbf{V}_p$ and $\mathbf{V}_n$ are the positive and negative sequence voltage phasors, respectively; $\mathbf{a} = 1\angle 120°$; and $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$ are the three-phase unbalanced line-to-ground voltage phasors. The use of sequence components $\mathbf{V}_p$ and $\mathbf{V}_n$ poses some practical difficulties due to the associated complex algebra [8] and the need to track both the voltage magnitudes and phase angles for each phase. One of the major challenges in using the IEC definition is the inability to directly measure the sequence components using installed RMS meters as they provide no information about the relative phase angles of the voltages [9]. For low- and medium-voltage systems, IEC standard 61000-2-2 [4] requires that the voltage unbalances, as defined in (1), are less than 2%.

B. NEMA definition (LVUR)

Motors make up a large portion of the loads connected to distribution systems. Motor manufacturers use the NEMA definition of voltage unbalance [6], which is also referred to as the Line Voltage Unbalance Rate (LVUR). The NEMA definition calculates unbalance using the magnitudes of line-to-line voltages V_{ab}, V_{bc} and V_{ca}:

$$LVUR\,[\%] = \frac{\Delta V_L^{max}}{V_L^{avg}} \times 100, \tag{2}$$
$$\text{where}\ \ V_L^{avg} = \frac{V_{ab} + V_{bc} + V_{ca}}{3},$$
$$\Delta V_L^{max} = \max\{|V_{ab} - V_L^{avg}|, |V_{bc} - V_L^{avg}|, |V_{ca} - V_L^{avg}|\}.$$

Per NEMA MG-1 [6] and ANSI C84.I [12], power systems must be operated such that the maximum voltage unbalance, as defined in (2), does not exceed 3% under no-load conditions.

C. IEEE definition (PVUR)

The IEEE definition is a recommended guideline to measure unbalance for the electrical design of industrial facilities [7]. It is commonly referred to as Phase Voltage Unbalance Rate (PVUR) and is described using the line-to-ground voltage magnitudes V_a, V_b and V_c:

$$PVUR\,[\%] = \frac{\Delta V_P^{max}}{V_P^{avg}} \times 100, \tag{3}$$
$$\text{where}\ \ V_P^{avg} = \frac{V_a + V_b + V_c}{3},$$
$$\Delta V_P^{max} = \max\{|V_a - V_P^{avg}|, |V_b - V_P^{avg}|, |V_c - V_P^{avg}|\}.$$

Motors are affected by phase-voltage unbalances that exceed 2% and are most likely to overheat when operating near full load [7]. By ignoring the phase angle information, the IEEE definition does not detect voltage unbalance that is due to phase angle asymmetries.

III. COMPARISON OF VOLTAGE UNBALANCE DEFINITIONS

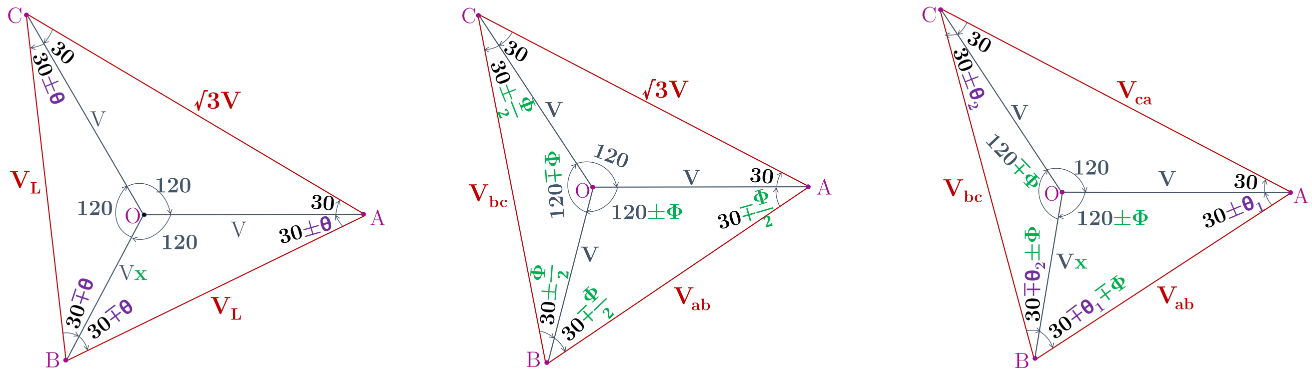
This section provides an in-depth comparison of the different voltage unbalance definitions by deriving analytical bounds that are verified through numerical simulations. To facilitate our analysis, we divide voltage unbalance into three categories:

(a) Voltage magnitude unbalance,
(b) Voltage angle unbalance,
(c) Voltage magnitude and angle unbalance.

Considering phase A as the reference phase, we analyze the impacts of varying the voltage phasors associated with either phase B alone or both phases B and C. Fig. 1 on the next page illustrates the voltage triangle with unbalance in phase B for each of these categories. With balanced voltage magnitude indicated by V, the magnitude unbalance is denoted using the multiplicative factor x and the angle unbalance is denoted using ϕ. The angle θ in Fig. 1 is determined by x and ϕ.

Starting with the simplest voltage unbalance cases involving only magnitude or angle unbalance, we utilize small angle approximations to derive analytical expressions for VUF, PVUR and LVUR. We then use the analytical expressions to characterize the relationships and derive bounds on the maximum difference among the various voltage definitions. In a second step, the validity of the bounds is tested using numerical simulations without any approximations involved.

In the more general cases with either angle unbalance in more than one phase or combinations of magnitude and angle



(a) Voltage magnitude unbalance (b) Voltage angle unbalance (c) Voltage magnitude and angle unbalance

Fig. 1: Voltage triangles for three cases of unbalance in phase B

unbalance, the analytical expressions become quite complex, and hard to analyze and compare. Hence, we refrain from including analytical expressions for these cases and instead rely on numerical simulations alone.

A. Voltage magnitude unbalance in phase B

This section investigates the voltage magnitude unbalance case (i.e., $\phi = 0°$). We first consider a voltage magnitude unbalance in one of the three phases as shown in Fig. 1(a). Two line-to-line voltages V_L are affected by the unbalance of the phase-B-to-ground voltage magnitude, while the third line-to-line voltage remains the same as in the balanced case with a magnitude of $\sqrt{3}V$. The line-to-ground and line-to-line voltage phasors are

$$\begin{aligned} \mathbf{V}_a &= V\angle 0°, & \mathbf{V}_{ab} &= V_L\angle(30° \pm \theta), \\ \mathbf{V}_b &= Vx\angle{-120°}, & \mathbf{V}_{bc} &= V_L\angle{-(90° \pm \theta)}, \\ \mathbf{V}_c &= V\angle 120°, & \mathbf{V}_{ca} &= \sqrt{3}V\angle 150°. \end{aligned}$$

Analytical Comparison We start by providing analytical expressions for the three voltage unbalance definitions.

1) IEC definition (VUF): The magnitudes of the positive and negative symmetrical components V_p and V_n are

$$V_p = \frac{|V\angle 0° + Vx\angle 0° + V\angle 0°|}{3} = \frac{V \cdot (x+2)}{3}, \tag{4a}$$

$$V_n = \frac{|V\angle 0° + Vx\angle 120° + V\angle{-120°}|}{3} = \frac{V\cdot(1-x)}{3}\frac{|1-j\sqrt{3}|}{2} = \frac{V\cdot|x-1|}{3}. \tag{4b}$$

The voltage unbalance according to the IEC definition is then

$$VUF = |x-1|/(x+2). \tag{5}$$

2) NEMA definition (LVUR): The LVUR voltage unbalance definition (2) depends on the line-to-line voltage V_L. To express V_L in terms of x, the sine triangle rule is applied to the triangle $\triangle OAB$ in Fig. 1(a):

$$\frac{V_L}{\sin 120°} = \frac{V}{\sin(30° \mp \theta)} = \frac{Vx}{\sin(30° \pm \theta)}. \tag{6}$$

Using (6) and the trigonometric angle sum identities yields two equations for V_L in terms of x and θ:

$$V_L \sin(30° \mp \theta) = V \sin 120° \Rightarrow V_L\Big(\cos\theta \mp \sqrt{3}\sin\theta\Big) = \sqrt{3}V, \tag{7a}$$

$$V_L \sin(30° \pm \theta) = Vx \sin 120° \Rightarrow V_L\Big(\cos\theta \pm \sqrt{3}\sin\theta\Big) = \sqrt{3}Vx. \tag{7b}$$

By adding (7a) to (7b) and applying the small angle approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, we obtain the following relation between V_L and x,

$$V_L = \sqrt{3}V\left(\frac{x+1}{2\cos\theta}\right) \approx \sqrt{3}V\left(\frac{x+1}{2}\right). \tag{8}$$

The average line-to-line voltage V_L^{avg} and maximum deviation ΔV_L^{max} are

$$V_L^{avg} = \frac{V_L + V_L + \sqrt{3}V}{3} \approx \sqrt{3}V\left(\frac{x+2}{3}\right), \tag{9a}$$

$$\Delta V_L^{max} = |\sqrt{3}V - V_{Lavg}| \approx \sqrt{3}V\left(\frac{|x-1|}{3}\right), \tag{9b}$$

which allows us to obtain the following expression for LVUR:

$$LVUR \approx |x-1|/(x+2). \tag{10}$$

3) IEEE definition (PVUR): The PVUR definition (3) is computed using the average voltage magnitude, V_P^{avg},

$$V_P^{avg} = \frac{V+V+Vx}{3} = \frac{V\cdot(x+2)}{3},$$

and the deviation from the average, ΔV_P^{max},

$$\Delta V_P^{max} = |Vx - \frac{V\cdot(x+2)}{3}| = \frac{2V\cdot|x-1|}{3}.$$

This gives rise to the following expression for PVUR,

$$PVUR = 2\cdot|x-1|/(x+2). \tag{11}$$

Comparing the three definitions based on (5), (10), and (11), we observe the following relationships:

$$LVUR \approx VUF = PVUR/2. \tag{12}$$

As indicated by the $\approx$ sign, the calculations of LVUR involve small angle approximations, whereas VUF and PVUR are exact calculations.

We observe that for a given deviation $|x-1|$ in voltage magnitude, the expressions for V_n, ΔV_L^{max} and ΔV_P^{max} in the numerators of (1), (2) and (3), respectively, are invariant to over-voltage ($x > 1$) or under-voltage ($x < 1$) conditions. Conversely, the expressions for V_p, V_L^{avg} and V_P^{avg} in the denominators of (1), (2) and (3), respectively, are directly proportional to x. Hence, the unbalance is greater for under-voltage cases compared to the over-voltage cases, which is consistent with the observations in [10].

Numerical Validation To validate the expressions derived above, we compute the voltage unbalances that result from varying the phase-B voltage magnitude. The multiplicative factor x is varied from 0.8 to 1.2 in steps of 0.01 to investigate both under-voltage and over-voltage conditions. The left part of Fig. 2 illustrates the variation of VUF (on the y-axis) with respect to both PVUR and LVUR (on the x-axis) for voltage magnitude unbalance in phase B only. The dashed black line represents the situations where VUF and LVUR have the same value, while the dashed red line represents the situation where PVUR is a factor of 2 larger than VUF. The orange and green dots correspond to values obtained for LVUR and PVUR, respectively.

Consistent with our analytical results, we observe that PVUR is exactly twice as large as VUF. The values for VUF and LVUR are almost identical, although there are minor variations due to the inaccuracy of the small angle approximation. For the range of x values considered, the relative difference $\left(\frac{|\text{LVUR}-\text{VUF}|}{\text{VUF}}\right)$ is less than 2%.

B. Voltage magnitude unbalance in phases B and C

We express the voltage unbalance in a similar way as the voltage unbalance for just phase B, except that we consider two deviations in the magnitudes, i.e., $V_b = x_1 V$ and $V_c = x_2 V$.

Analytical Comparisons Due to the complexity of the expressions, we only include a high-level discussion of the case with

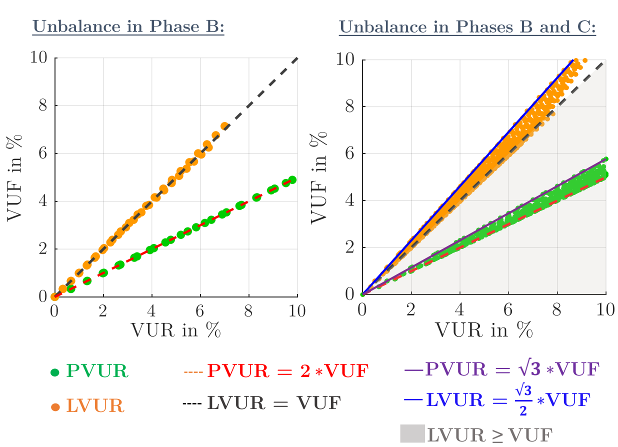


Fig. 2: Voltage magnitude unbalance for $0.80 \le x \le 1.20$

voltage magnitude unbalances in both phases B and C. Similar to the case with magnitude deviation in only one phase, the denominators of all unbalance definitions simplify to the same expressions when we have magnitude deviations in two phases. To assess the relative magnitudes of the voltage unbalance definitions for a given voltage unbalance, we therefore only need to assess the differences in the numerators of (1), (2) and (3) .

1) IEC definition (VUF): The numerator for VUF in (1) is the magnitude of the negative sequence voltage V_n:

$$|\mathbf{V}_n| = \frac{V}{3}\sqrt{x_1^2 + x_2^2 + 1 - x_1 - x_2 - x_1 x_2}. \qquad (13)$$

2) NEMA and IEEE definitions (LVUR and PVUR): The numerators for LVUR and PVUR in (2) and (3), respectively, depend on which voltage magnitude is furthest from the average. We hence need to consider three cases,

$$\Delta V_L^{max} \approx \frac{\Delta V_P^{max}}{2} = \frac{V}{6}\max\{|y_1|, |y_2|, |y_3|\}, \qquad (14)$$

$$\text{where} \quad y_1 = x_1 - 2x_2 + 1, \quad y_2 = x_1 + x_2 - 2, \quad y_3 = -2x_1 + x_2 + 1$$

Comparing the three definitions, we observe that the denominator for LVUR is still approximately equivalent to the denominator for PVUR, suggesting the following relationship between PVUR and LVUR:

$$\frac{PVUR}{LVUR} \approx \frac{\Delta V_P^{max}}{\Delta V_L^{max}} \approx 2 \;\Rightarrow\; PVUR \approx 2 \cdot LVUR. \qquad (15)$$

When comparing LVUR and PVUR with VUF, we observe that there is not a one-to-one mapping because we need to consider the three different cases corresponding to whether y_1, y_2, or y_3 is the largest. Since both $|\mathbf{V}_n|$ and ΔV_P^{max} are non-negative, we consider the square of the ratio between VUF and PVUR, i.e.,

$$\left(\frac{VUF}{PVUR}\right)^2 = \frac{|\mathbf{V}_n|^2}{(\Delta V_P^{max})^2} = \frac{x_1^2 + x_2^2 + 1 - x_1 - x_2 - x_1 x_2}{(\max\{|y_1|, |y_2|, |y_3|\})^2}.$$

By identifying the largest among $|y_1|, |y_2|$ and $|y_3|$, simplifying the expressions, and taking the square root of both sides, we obtain an upper bound on the ratio of VUF to PVUR,

$$\frac{|\mathbf{V}_n|}{\Delta V_P^{max}} \le \frac{1}{\sqrt{3}} \quad \Rightarrow \; VUF \le \left(\frac{1}{\sqrt{3}}\right) \cdot PVUR. \qquad (16)$$

The definitions of y_1, y_2, and y_3 imply that the unbalance ratio seen in (16) with unbalance in two phases is always larger than the situation where either $x_1 = 1$ or $x_2 = 1$. This implies that the case with magnitude unbalance only in phase B (12) is a lower bound on the more general case with magnitude unbalance in two phases. Combining this observation with (16) yields the following relationship between PVUR and VUF:

$$\left(\frac{1}{2}\right) \cdot PVUR \le VUF \le \left(\frac{1}{\sqrt{3}}\right) \cdot PVUR. \qquad (17a)$$

By considering (15), we obtain an approximate upper bound on the ratio of VUF to LVUR,

$$LVUR \lesssim VUF \lesssim \left(\frac{2}{\sqrt{3}}\right) \cdot LVUR \tag{17b}$$

As indicated by the $\lesssim$ sign, the calculations of ΔV_L^{max} (numerator of LVUR) in (14) involves small angle approximations.

Numerical Validation Similar to the case with unbalance in only one phase, we run experiments where we let x_1 and x_2 vary from 0.8 to 1.2 in steps of 0.01. The results are shown in the right part of Fig. 2 with a similar legend as the left part. In addition to the black and red lines that are used to identify the situation where $LVUR = VUF$ and $PVUR = 2 \cdot VUF$ respectively, we also add lines to represent the upper bounds $VUF = (\frac{2}{\sqrt{3}}) \cdot LVUR$ (solid blue line) and $VUF = (\frac{1}{\sqrt{3}}) \cdot PVUR$ (solid purple line). Almost all points fall within the predicted range.

However, LVUR is greater than VUF for approximately 17% of the considered points. These points fall below the dashed black line in the grey shaded region as shown in Fig. 2. This situation corresponds to cases where two of the line-to-line voltages were significantly higher than the third line-to-line voltage, i.e., extreme cases of under-voltage in two phases and over-voltage in the third phase, resulting in inaccuracy of the small angle approximation. However, the errors in our bounds are small: within the ranges of voltage magnitudes corresponding to $0.8 \leq x_1, x_2 \leq 1.2$, we never see violations larger than 2% relative to the bounds in (17).

C. Voltage angle unbalance in phase B

We next consider the single voltage angle unbalance case with angle displacement ϕ and balanced voltage magnitude V as shown in Fig. 1(b). The voltage phasors in this case are

$$\begin{aligned} &\mathbf{V}_a = V\angle 0^\circ, && \mathbf{V}_{ab} = V_{ab}\angle(30^\circ \mp \phi/2),\\ &\mathbf{V}_b = V\angle{-(120^\circ \pm \phi)}, && \mathbf{V}_{bc} = V_{bc}\angle{-(90^\circ \pm \phi/2)},\\ &\mathbf{V}_c = V\angle 120^\circ, && \mathbf{V}_{ca} = \sqrt{3}V\angle 150^\circ. \end{aligned}$$

Analytical comparison

1) IEC definition (VUF): To derive the expression for VUF, we use the small angle approximation to find the magnitudes of the sequence voltages, V_p and V_n:

$$V_p \approx \frac{V}{3}\sqrt{9+\phi^2} \approx V, \quad V_n \approx V \cdot \frac{\phi}{3} \quad \Rightarrow \quad VUF \approx \frac{\phi}{3}. \tag{18}$$

2) NEMA definition (LVUR): To determine LVUR, we use the sine triangle rule and the small angle approximation for ϕ to obtain the line-to-line voltage magnitudes, V_{ab} and V_{bc}:

$$V_{ab} \approx V(\sqrt{3} \pm \frac{\phi}{2}), \qquad V_{bc} \approx V(\sqrt{3} \mp \frac{\phi}{2}). \tag{19}$$

We then derive approximations of the average line-to-line voltage V_L^{avg} and the maximum deviation ΔV_L^{max}, which provide an approximate expression for LVUR:

$$V_L^{avg} \approx \sqrt{3}V, \;\; \Delta V_L^{max} \approx V\frac{\phi}{2} \;\; \Rightarrow \;\; LVUR \approx \frac{\sqrt{3}}{2}\frac{\phi}{3}. \tag{20}$$

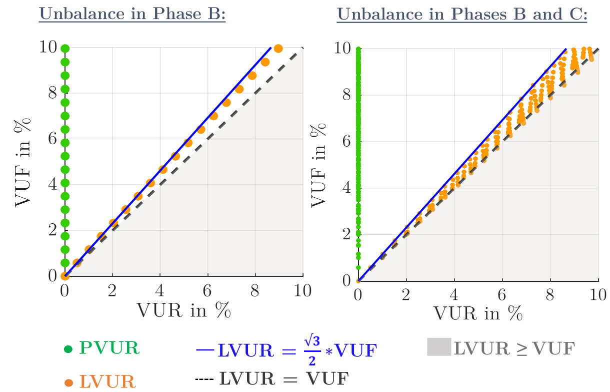


Fig. 3: Voltage angle unbalance for -20° $\leq \phi \leq$ 20°

3) IEEE definition (PVUR): Since the line-to-ground voltage magnitudes are balanced, $\Delta V_P^{max} = 0$. Hence, $PVUR = 0$ for any unbalance that only affects the voltage angles.

Based on the analytical expressions above, we observe that PVUR does not provide any information about phase angle unbalance. The approximate relationship between LVUR and VUF is

$$LVUR \lesssim VUF \lesssim \left(\frac{2}{\sqrt{3}}\right) \cdot LVUR. \tag{21}$$

Numerical Validation The relationship in (21) is verified by numerical experiments. With phase A selected as a reference, the phase angle displacement in phase B is varied in the range $-20^\circ \leq \phi \leq 20^\circ$ in steps of 1°. The results of these computations are shown in the left part of Fig. 3 for unbalance in phase B. The experiment confirms our the analytical results, with $PVUR = 0$ regardless of the angle unbalance and $LVUR \approx (\frac{\sqrt{3}}{2}) \cdot VUF$.

D. Voltage angle unbalance in Phases B and C

To extend the above analysis, we assess how voltage angle unbalance in two phases affects the relationships among the definitions. We introduce two phase angle deviations, ϕ_1 and ϕ_2 for phases B and C, respectively. Due to the complexity of the resulting analytical expressions, we only detail the numerical results for this case.

Numerical Validation We perform a similar numerical study as above, but consider angle unbalance in both phases B and C. We use phase A as a reference, and consider all combinations of phase angle deviations in phases B and C in the range $-20^\circ \leq \phi_1, \phi_2 \leq 20^\circ$, in steps of 1°. The results are shown in the right part of Fig. 3. We observe that PVUR, which includes no information about phase angle unbalance, is still always zero for all combinations of angle unbalance.

As with voltage magnitude unbalance, introducing phase angle unbalance in both phases means that there is no longer a direct correspondence between LVUR and VUF. Interestingly, we can numerically verify that VUF is still effectively bounded by the same relations as seen in (17b) or (21).

E. Voltage magnitude and angle unbalance

Finally, we consider the general case where the unbalance involves asymmetries in both the voltage magnitudes and angles. We first investigate the case with both magnitude and angle unbalance in phase B, which has an unbalanced voltage magnitude Vx and the angle displacement defined by ϕ as shown in Fig. 1(c) and expressed below:

$$\begin{aligned} \mathbf{V}_a &= V\angle 0^\circ, & \mathbf{V}_{ab} &= V_{ab}\angle(30^\circ \mp \theta_1), \\ \mathbf{V}_b &= Vx\angle{-(120^\circ \pm \phi)}, & \mathbf{V}_{bc} &= V_{bc}\angle{-(90^\circ \pm \theta_2)}, \\ \mathbf{V}_c &= V\angle 120^\circ, & \mathbf{V}_{ca} &= \sqrt{3}V\angle 150^\circ. \end{aligned}$$

In addition, we consider the situation where we have similarly defined unbalances for both magnitudes and angles in both phases B and C. Due to the complexity of the expressions, we derive our bounds based on numerical validation only.

Numerical Comparison To perform our numerical experiment, we choose $\mathbf{V}_a$ as reference voltage phasor at $1\angle 0^\circ$ and calculate the voltage unbalance according to each definition as we vary the deviations in both the voltage magnitudes (from 0.8 to 1.2 in steps of 0.01) and angles (from -20° to 20° in steps of 1°). The results are shown in Fig. 4. The left part of this figure shows the case with deviations only in phase B and the right part depicts the case with unbalances in both phases B and C.

As previously discussed, PVUR does not consider voltage angle unbalances. Hence, Fig. 4 shows that there is a large variation of PVUR corresponding to any given value of VUF.

The relationship between LVUR and VUF is similar to the previous cases, yielding the same bounds as in (17b) or (21). The few points where $VUF < LVUR$ correspond to the extreme condition of under-voltage in two phases and over-voltage in the third phase. For the ranges of voltage magnitude deviations $0.8 \leq x_1, x_2 \leq 1.2$ and phase angle deviations $-20^\circ \leq \phi_1, \phi_2 \leq 20^\circ$, we never observe violations greater than 5% relative to the bounds in (17b).

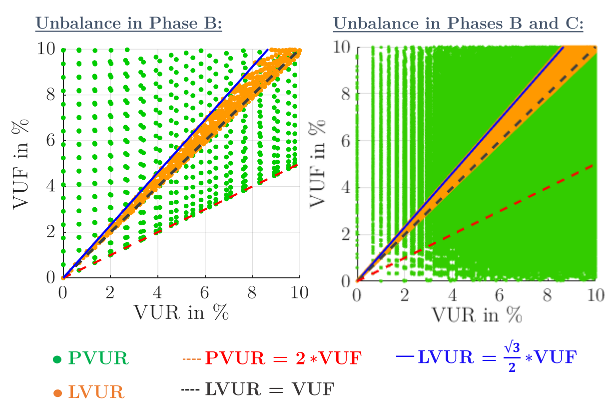


Fig. 4: Voltage magnitude and angle unbalances for $0.8 \leq x \leq 1.2$ and $-20^\circ \leq \phi \leq 20^\circ$.

IV. Conclusion

This paper analyzes the relationships among the three most prevalent definitions of voltage unbalance. The "true" voltage unbalance definition used by the IEC (VUF) is based on negative and positive sequence voltages. Thus, VUF captures both voltage magnitude and angle unbalance, but can be difficult to measure because it requires measurements of the relative phase angles. Definitions from NEMA and IEEE based on the line-to-line voltages (LVUR) and line-to-ground voltages (PVUR), respectively, are frequent substitutes used in the design of electrical machines and industrial plants. The goal of our paper is to extend existing comparisons among these definitions with a more comprehensive analysis that considers a range of voltage unbalance conditions, using both analytical derivations and numerical simulations.

To this end, we derive and numerically demonstrate relationships among the different definitions. In summary, PVUR does not provide any information about phase angle unbalance and has no clear relationship with VUF. Conversely, LVUR provides information about unbalances in both voltage magnitudes and phases. Using the small angle approximation, we bound the relationships between the IEC definition (VUF) and the NEMA definition (LVUR) as

$$LVUR \lesssim VUF \lesssim \left(\frac{2}{\sqrt{3}}\right) \cdot LVUR.$$

For a reasonable ranges of voltage magnitude and angle deviations (0.8 to 1.2 p.u. voltage magnitude and -20° to 20° voltage angle), the violations of these bounds are less than 5% (relative to the bounds).

Our ongoing work is investigating how these definitions can be formulated in order to provide the most tractable constraints in three-phase optimal power flow problems that include minimization of voltage unbalance.

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