Assessing the Accuracy of Balanced Power System Models in the Presence of Voltage Unbalance

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***Abstract*—Traditional models of electric power systems represent distribution systems with unbalanced three-phase network models and transmission systems with balanced single-phase-equivalent network models. This distinction poses a challenge for coupled models of transmission and distribution systems, which are becoming more prevalent due to the growth of distributed energy resources connected to distribution systems. In order to maintain a balanced network representation, transmission system models typically assume that the voltage phasors at the interface to the distribution system are balanced. Inaccuracies resulting from this assumption during unbalanced operation can lead to erroneous values for line currents in the transmission system model. This paper empirically quantifies the accuracy of this balanced operating assumption during unbalanced operating conditions for both a simple two-bus system along with a more complex transmission and distribution co-simulation. This paper also characterizes the performance of different methods for translating the unbalanced voltage phasors into a balanced representation in order to give recommendations for modeling coupled transmission and distribution systems.**

***Index Terms*—power distribution, power transmission, renewable energy**

I. Introduction

Power systems are designed and operated to satisfy strict requirements with respect to voltage balance; i.e., the voltage phasors at each bus in a three-phase system must be approximately equal in magnitude and offset by approximately 120°. System operators typically achieve adequately balanced voltages by infrequently (e.g., seasonally) reconnecting loads to different phases. Sole reliance on this traditional approach is challenged by the variability inherent to renewable energy resources [1]. Distributed energy resources (DERs) and virtual renewable power plants (VRPPs) can exacerbate voltage unbalance problems [2]–[4]. Voltage unbalance increases losses and poses reliability challenges, particularly with respect to three-phase induction motors [5]. The negative impacts from voltage unbalance are estimated to cost U.S. industries up to $28 billion annually [6]. Therefore, managing and preventing unbalanced operating conditions is of high importance for reliable and efficient power system operation.

The impact of unbalanced conditions on the operation and management of power systems is a pivotal problem to understand and address. Synchronous generators connected to the transmission system are rapidly being replaced with VRPPs and DERs, especially rooftop solar, in distribution feeders. While the challenges that unbalanced operation poses to synchronous generators are known and should be avoided, the technical ramifications of operating power electronic inverters under unbalanced conditions are less well understood. A review of the literature suggests that the problem of load and voltage unbalance has been primarily studied in distribution systems [2]–[4], [7]–[10] and for transmission systems, it has been chiefly studied with a focus on impacts to power flow [11], [12].

This work analyzes the impacts of unbalanced operating conditions from the distribution system propagating onto the transmission system. Power systems are hierarchical dynamic systems with the transmission system and distribution system being two distinct yet interdependent layers in this hierarchy. Due to the interconnected nature of power systems, unbalanced conditions can propagate between distribution and transmission systems. Temporarily unbalanced operation due to faults has been studied computationally in [13]. For co-simulation, strategies that model the three sequence components of the transmission system have been proposed [14]; however, co-simulations commonly assume a balanced transmission system model (positive sequence only) [15].

This paper does not specifically address whether a balanced model of the transmission system is an appropriate approximation relative to endogenously modeling unbalances in the transmission system. Rather, this paper compares different options for interfacing a balanced (single-phase equivalent) transmission system model with unbalanced (three-phase) distribution system models. Complementing prior literature, this paper's main contributions are:

1) We empirically characterize the impacts of the balanced voltage assumption used in traditional transmission network models by considering varying levels of unbalance according to different unbalance definitions.
2) Based on this analysis, we compare methods for interfacing unbalanced three-phase distribution network models with balanced single-phase-equivalent transmission network models.

The remainder of this paper is organized as follows. Section II describes metrics for quantifying unbalance, methods for generating balanced sets of phasors, and a metric for current flow error. Section III presents the setup and results for a two-bus case study. Section IV extends the analysis to a co-simulation on the nine-bus WECC system combined with representative distribution feeder models. Section V summarizes the key results, conclusions, and recommendations for future work.

II. Quantifying and Analyzing Unbalance

This section discusses several standard metrics used to measure phase unbalance in three-phase AC electric power systems. Section II-A describes the voltage unbalance metrics used in various power quality standards. Section II-B defines four approaches for constructing balanced voltage phasors from a set of unbalanced voltage phasors. Finally, Section II-C defines a metric for characterizing the current unbalance, which is used as the main output considered in our analyses.

A. Voltage Unbalance Metrics

The three most common definitions of voltage unbalance appear in 1) the IEEE standard [16], 2) the NEMA standard [17], and 3) the IEC standard [18]. These definitions are based on either the line-to-ground voltages, $\boldsymbol{V_a} = |V_a|\angle\theta_a$, $\boldsymbol{V_b} = |V_b|\angle\theta_b$, and $\boldsymbol{V_c} = |V_c|\angle\theta_c$, or line-to-line voltages, $\boldsymbol{V_{ab}} = \boldsymbol{V_a} - \boldsymbol{V_b}$, $\boldsymbol{V_{bc}} = \boldsymbol{V_b} - \boldsymbol{V_c}$, and $\boldsymbol{V_{ca}} = \boldsymbol{V_c} - \boldsymbol{V_a}$. Prior research provides analytical [7] and numerical [19], [20] comparisons of various voltage unbalance standards.

IEEE definition: This definition is based on line-to-ground voltage magnitudes [16]:

$$U_{IEEE}\ [\%] = \frac{\Delta V_{lg}^{max}}{V_{lg}^{avg}} \times 100, \tag{1}$$

where

$$V_{lg}^{avg} = \frac{|V_a| + |V_b| + |V_c|}{3},$$
$$\Delta V_{lg}^{max} = \max\{||V_a| - V_{lg}^{avg}|, ||V_b| - V_{lg}^{avg}|, ||V_c| - V_{lg}^{avg}|\}.$$

According to [16], U_{IEEE} should not exceed 2% in order to avoid motor overheating.

NEMA definition: This definition is based on the line-to-line voltage magnitudes [17]:

$$U_{NEMA}\ [\%] = \frac{\Delta V_{ll}^{max}}{V_{ll}^{avg}} \times 100, \tag{2}$$

where

$$V_{ll}^{avg} = \frac{|V_{ab}| + |V_{bc}| + |V_{ca}|}{3},$$
$$\Delta V_{ll}^{max} = \max\{||V_{ab}| - V_{ll}^{avg}|, ||V_{bc}| - V_{ll}^{avg}|, ||V_{ca}| - V_{ll}^{avg}|\}.$$

The NEMA standard MG-1 requires U_{NEMA} to remain within 3% [17].

IEC definition: This definition is based on the sequence component transformation [21]. Define $\boldsymbol{\alpha} = 1\angle 120°$. The IEC definition of phase unbalance is [18]:

$$U_{IEC}\ [\%] = \frac{|V_n|}{|V_p|}, \tag{3}$$

where

$$\boldsymbol{V_p} = \frac{\boldsymbol{V_a} + \boldsymbol{\alpha} \cdot \boldsymbol{V_b} + \boldsymbol{\alpha}^2 \cdot \boldsymbol{V_c}}{3},$$
$$\boldsymbol{V_n} = \frac{\boldsymbol{V_a} + \boldsymbol{\alpha}^2 \cdot \boldsymbol{V_b} + \boldsymbol{\alpha} \cdot \boldsymbol{V_c}}{3}.$$

In (3), $\boldsymbol{V_p}$ and $\boldsymbol{V_n}$ are the positive and negative sequence components, respectively. We denote the angle of $\boldsymbol{V_p}$ as θ_p. The IEC standard 61000-2-2 requires U_{IEC} to be less than 2% [18].

IEC Adapted Standard: As an alternative, we also considered the following modification to the IEC definition that considers the positive, negative, and zero sequence components:

$$U_{IEC,pnz}\ [\%] = \max\left\{\frac{|V_n|}{|V_p|}, \frac{|V_z|}{|V_p|}\right\}, \tag{4}$$

where $\boldsymbol{V_n}, \boldsymbol{V_p}$ are defined in (3) and $\boldsymbol{V_z} = \frac{\boldsymbol{V_a}+\boldsymbol{V_b}+\boldsymbol{V_c}}{3}$. Intuitively, this definition will reflect high levels of unbalance for the cases with a large zero sequence component.

B. Constructing Balanced Voltage Phasors

Coupling a balanced transmission network model with unbalanced distribution system models requires an approach for interfacing the balanced voltage phasors in the former model with the unbalanced voltage phasors in the latter model. We denote the balanced voltage phasors as $\boldsymbol{\tilde{V}_a} = |\tilde{V}_a|\angle\tilde{\theta}_a$, $\boldsymbol{\tilde{V}_b} = |\tilde{V}_a|\angle(\tilde{\theta}_a - 120°)$, and $\boldsymbol{\tilde{V}_c} = |\tilde{V}_a|\angle(\tilde{\theta}_a + 120°)$. We describe four methods for computing $\boldsymbol{\tilde{V}_a}$, $\boldsymbol{\tilde{V}_b}$, and $\boldsymbol{\tilde{V}_c}$ from a set of unbalanced voltage phasors $\boldsymbol{V_a} = |V_a|\angle\theta_a$, $\boldsymbol{V_b} = |V_b|\angle\theta_b$, and $\boldsymbol{V_c} = |V_c|\angle\theta_c$

Positive Sequence Method: This approach simply uses the positive sequence component of the unbalanced voltages as the balanced phasors:

$$\boldsymbol{\tilde{V}_a} = |V_p|\angle\theta_p, \tag{5a}$$
$$\boldsymbol{\tilde{V}_b} = |V_p|\angle(\theta_p - 120°), \tag{5b}$$
$$\boldsymbol{\tilde{V}_c} = |V_p|\angle(\theta_p + 120°), \tag{5c}$$

where $|V_p|$ and θ_p are the magnitude and angle, respectively, of the positive sequence voltage phasor defined in (3). This approach is common in the literature for co-simulation studies that assume a balanced transmission system model [15].

Match Phase A Angle Method: This approach uses the average magnitude from the unbalanced voltage phasors (V_{lg}^{avg} defined in (1)) and the phase A voltage angle (θ_a) to set the voltage magnitudes and angles of the balanced phasors:

$$\boldsymbol{\tilde{V}_a} = V_{lg}^{avg}\angle\theta_a, \tag{6a}$$
$$\boldsymbol{\tilde{V}_b} = V_{lg}^{avg}\angle(\theta_a - 120°), \tag{6b}$$
$$\boldsymbol{\tilde{V}_c} = V_{lg}^{avg}\angle(\theta_a + 120°). \tag{6c}$$

By definition, this method matches the Phase A angle exactly, while any errors due to unbalance are seen in the remaining phases.

Minimize Angle Error Method: This approach is intended to incorporate more information regarding all three of the voltage

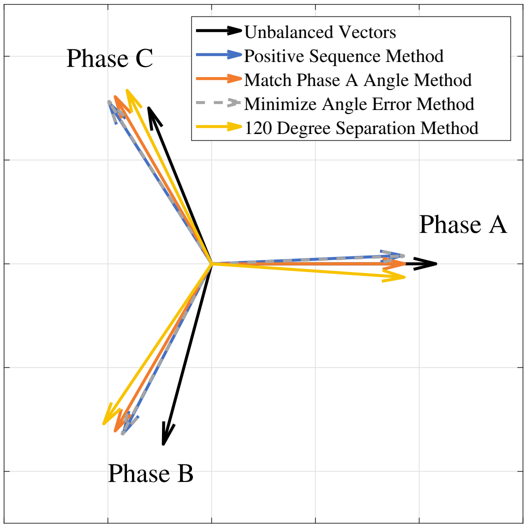

Fig. 1. Visual comparison of the four methods used to generate a balanced set of voltage phasors ($\tilde{V}_a$, $\tilde{V}_b$, and $\tilde{V}_c$) from a set of unbalanced voltage phasors (V_a, V_b, and V_c).

angles. We select the phase A voltage angle for the balanced voltage phasors, $\tilde{\theta}_a$, that leads to the minimum sum of the squared differences with respect to the unbalanced voltage angles, θ_a, θ_b, and θ_c. In other words, we choose $\tilde{\theta}_a$ to be the solution to the following regression problem:

$$\min_{\tilde{\theta}_a} \; (\theta_a-\tilde{\theta}_a)^2+(\theta_b-(\tilde{\theta}_a-120^\circ))^2+(\theta_c-(\tilde{\theta}_a+120^\circ))^2. \quad (7)$$

The solution to this regression problem is the average of θ_a, θ_b, and θ_c:

$$\tilde{\theta}_a = \frac{\theta_a + (\theta_b + 120^\circ) + (\theta_c - 120^\circ)}{3} = \frac{\theta_a+\theta_b+\theta_c}{3}. \quad (8)$$

Similar to (6), we set the magnitude of the balanced voltages to the average magnitude of the unbalanced voltage phasors:

$$\tilde{\boldsymbol{V}}_{\boldsymbol{a}} = V_{lg}^{avg}\angle\tilde{\theta}_a, \quad (9a)$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{b}} = V_{lg}^{avg}\angle(\tilde{\theta}_a - 120^\circ), \quad (9b)$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{c}} = V_{lg}^{avg}\angle(\tilde{\theta}_a + 120^\circ). \quad (9c)$$

120 Degree Separation Method: The final method centers the balanced phasor representation over the two unbalanced phasors whose difference is closest to 120°.

Define a three-element set S as:

$$S = \{(\theta_a-\theta_b)-120^\circ, (\theta_b-\theta_c)-120^\circ, (\theta_c-\theta_a)-120^\circ\}. \quad (10)$$

The elements of S indicate how close each pair of phasors is from a balanced 120° separation. For a balanced set of phasors, S is a set of zeros ($\{0^\circ, 0^\circ, 0^\circ\}$). Balanced voltage phasors are constructed by using the pair of unbalanced phasors with closest to 120° separation as the reference:

$$\begin{bmatrix}\tilde{\boldsymbol{V}}_{\boldsymbol{a}}\\ \tilde{\boldsymbol{V}}_{\boldsymbol{b}}\\ \tilde{\boldsymbol{V}}_{\boldsymbol{c}}\end{bmatrix} = \begin{cases} \begin{matrix} V_{lg}^{avg}\angle(\theta_a - \frac{S[i_{min}]}{2}) \\ V_{lg}^{avg}\angle(\theta_a - \frac{S[i_{min}]}{2} - 120^\circ), \\ V_{lg}^{avg}\angle(\theta_a - \frac{S[i_{min}]}{2} + 120^\circ) \end{matrix} & i_{min} = 1 \\[2ex] \begin{matrix} V_{lg}^{avg}\angle(\theta_b - \frac{S[i_{min}]}{2} + 120^\circ) \\ V_{lg}^{avg}\angle(\theta_b - \frac{S[i_{min}]}{2}), \\ V_{lg}^{avg}\angle(\theta_b - \frac{S[i_{min}]}{2} - 120^\circ) \end{matrix} & i_{min} = 2 \\[2ex] \begin{matrix} V_{lg}^{avg}\angle(\theta_c - \frac{S[i_{min}]}{2} - 120^\circ) \\ V_{lg}^{avg}\angle(\theta_c - \frac{S[i_{min}]}{2} + 120^\circ), \\ V_{lg}^{avg}\angle(\theta_c - \frac{S[i_{min}]}{2}) \end{matrix} & i_{min} = 3 \end{cases} \quad (11)$$

where i_{min} is the index of the smallest element of S. The goal of this method is to improve the accuracy of two phases by centering the balanced set on the pair of phasors with closest to 120° separation while sacrificing a larger angle difference for the last phase.

Fig. 1 shows the balanced sets of voltage phasors that result from applying the four methods to a representative unbalanced set. The vector magnitudes are similar for all four methods. The angles, however, can vary widely. In the example shown, the Positive Sequence Method and the Minimize Error Angle Method have a very similar but not identical result. For the 120 Degree Separation Method, phases A and C have the closest to 120° separation and it can be seen that the balanced phasors are centered around phases A and C.

C. Current Unbalance Metric

In order to compare a network model that assumes balanced voltages relative to an unbalanced three-phase model, we employ the following metric to quantify the error in the balanced system relative to the unbalanced system. The current flows corresponding to the voltage phasors at a pair of buses m and n ($\boldsymbol{V_{m,a}}$, $\boldsymbol{V_{m,b}}$, $\boldsymbol{V_{m,c}}$ and $\boldsymbol{V_{n,a}}$, $\boldsymbol{V_{n,b}}$, $\boldsymbol{V_{n,c}}$, respectively) connected by a line with per phase impedance $\boldsymbol{Z_{line}}$ are given by Ohm's law:

$$\boldsymbol{I_{line,\phi}} = \frac{\boldsymbol{V_{m,\phi}} - \boldsymbol{V_{n,\phi}}}{\boldsymbol{Z_{line}}}, \qquad \forall\phi = \{a, b, c\}. \quad (12)$$

The balanced network assumption results in balanced current flows that are similarly computed using Ohm's law:

$$\tilde{\boldsymbol{I}}_{\boldsymbol{line,\phi}} = \frac{\tilde{\boldsymbol{V}}_{\boldsymbol{m,\phi}} - \tilde{\boldsymbol{V}}_{\boldsymbol{n,\phi}}}{\boldsymbol{Z_{line}}}, \qquad \forall\phi = \{a, b, c\}. \quad (13)$$

To characterize the impacts of the balanced voltage phasor assumption, we calculate the error, ρ, between the balanced and unbalanced current flows as the maximum of the errors for each phase (denoted as ρ_a, ρ_b, and ρ_c):

$$\rho \text{ [p.u.]} = \max\{\rho_a, \rho_b, \rho_c\}, \quad (14)$$

where

$$\rho_\phi = |(\tilde{\boldsymbol{I}}_{\boldsymbol{line,\phi}} - \boldsymbol{I_{line,\phi}})|, \quad \phi \in \{a, b, c\}.$$

While it is also possible to calculate the average error over the three phases, the maximum error is likely a better indicator of problematic operating conditions in many practical applications.

III. Case Study: Two-Bus Test System

This section describes the two-bus test system, the methods for generating unbalanced voltage phasors, and the current error results across a range of voltage unbalance definitions, balancing methods, and levels of unbalance.

A. System Description

We first consider the simple two-bus transmission system model shown in Fig. 2. For this system, the line impedance is $\boldsymbol{Z_{line}} = 0.05 + j0.20$ per unit. Unbalanced loading in the distribution systems connected at buses m and n leads to unbalanced voltage phasors at these buses.

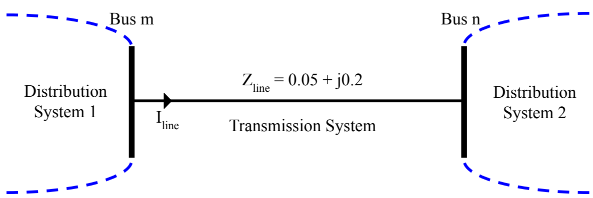

Fig. 2. Two-bus test system.

B. Generating Sets of Unbalanced Phasors

To study the impacts of voltage unbalance on this system, we generate sets of random unbalanced voltage phasors for buses m and n. In a co-simulation environment, these unbalanced voltages would come from the power flow solutions for individual distribution systems; however, in this simplified case study, they are generated randomly in order to consider a wide variety of possible operating conditions. The voltage magnitudes are sampled randomly from a uniform distribution from 0.95 to 1.05 per unit in accord with standard voltage magnitude requirements [22]. The angles of phases B and C are also sampled randomly from a uniform distribution of -10° to 10° away from the balanced case relative to phase A, which is itself selected randomly from 0° to 360°. From this initial set of voltage phasors, only those with under 2% unbalance according to the unbalance metric of interest from Section II-A are kept for analysis.

The ranges used for sampling (0.95 to 1.05 p.u. and $\pm 10^\circ$) are selected since they routinely give phasors within our defined unbalance limits, thus limiting computational time during simulations compared to using larger ranges. Moreover, the unbalance definitions usually give similar values for voltage phasors sampled within this range and are also expected to be qualitatively similar to the unbalances seen in typical distribution systems. We therefore believe that the conclusions we draw from the results in the remainder of this section are applicable to practical power systems. This is further verified in the larger case study in Section IV. We note that sampling from significantly larger ranges can have non-negligible impacts on the results.

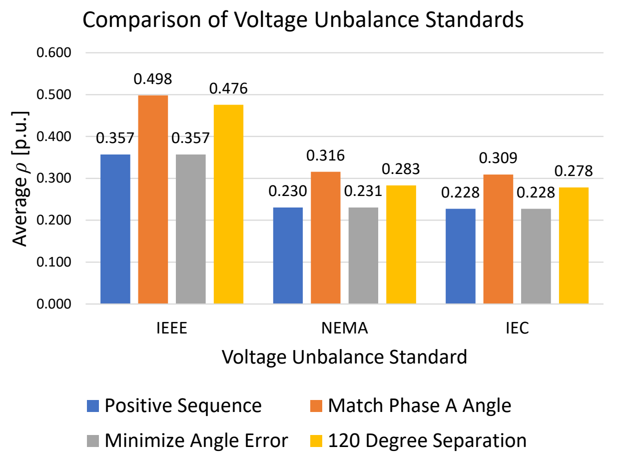

Fig. 3. Comparison of current flow errors resulting from randomly sampled voltage phasors with less than or equal to 2% unbalance according to various voltage unbalance definitions.

For each pair of three-phase voltage phasors that meet the unbalance limits ($\leq 2\%$), we compute the current flow error, ρ, as defined in (14). We consider all pairs of 1000 randomly sampled sets of unbalanced voltage phasors to compute approximately $500,000$ values for ρ. We repeat this process for each of the three voltage unbalance definitions (IEEE, NEMA, and IEC) and also for each of the four voltage balancing methods (Positive Sequence, Match Phase A Angle, Minimize Angle Error, and 120 Degree Separation). In the results that follow, we examine the distribution and average values for the error ρ across the various definitions and balancing methods.

To put the following error results in context, we note that the average current flow in the two-bus system from balanced voltage phasors sampled within the considered range is 6.2 pu.

C. Results

A summary of the average results are shown in Fig. 3. The IEEE definition results in the greatest average error, while the NEMA and IEC definitions are comparable, with the IEC definition resulting in slightly lower errors. For all unbalance definitions, the Positive Sequence Method and Minimize Angle Error Method give very similar results, both of which are superior to the results from the Match Phase A Angle Method and the 120 Degree Separation Method.

While the IEC definition of voltage unbalance results in the lowest average current flow error, this definition only considers the positive and negative sequence components, neglecting the zero sequence component. This can lead to large differences between the balanced and unbalanced current flows. Figure 4 shows the average results for the IEC and IEC Adapted Standards. The more strict IEC Adapted Standard removes voltage phasors with large zero sequence components from consideration, and thereby reduces the average current flow error by approximately a factor of two across all balancing methods.

Next, we analyze the distribution of the current error with respect to the level of unbalance in the voltage phasors. Fig. 5 shows a representative subset of the current flow errors versus their corresponding voltage unbalance. In this figure, the x-axis indicates the voltage unbalance value averaged over both of the line's terminal buses (m and n) and the y-axis corresponds to the error in the current flow (ρ). As expected, all methods for computing balanced voltage phasors tend to incur higher errors when the set of voltage phasors is more unbalanced. Qualitatively, it appears that there are more outliers with the Match Phase A Angle method compared to the other methods, especially at larger values of voltage unbalance (above 1.6%). The distribution of the error data is shown in Fig. 6, with the outlier data points shown explicitly. There are many more instances of high values of ρ for the Match Phase A Angle and 120 Degree Separation methods.

IV. Case Study: Nine-Bus Transmission and Distribution Co-simulation

The interface between transmission and distribution system models in a co-simulation environment is a common example of when the balancing assumptions described above are employed in practice. Therefore, to evaluate the methods in a more realistic scenario, we next show results from running a set of co-simulations with each balancing method described in Section II-B. In this section, we compare the errors in the resulting current flows.

A. System Description

The co-simulation system is shown in Fig. 7. The transmission system is the nine-bus WECC system with each fixed load in the nine-bus system replaced by one of three EPRI test feeders which are included with the OpenDSS simulator [23]. Table I gives a summary of the three feeders. The average real power unbalance on each phase is calculated by comparing the real power on each phase to the average power across all three phase and averaging over a one month period. The

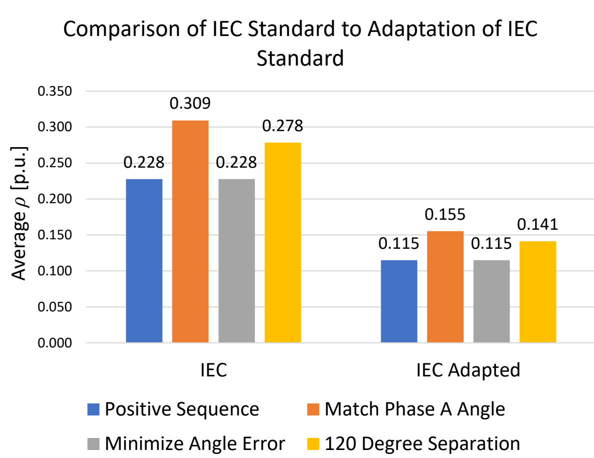

Fig. 4. Comparison of the current flow errors associated with the 2% voltage unbalance according to both the IEC definition (3) and the adapted IEC definition (4).

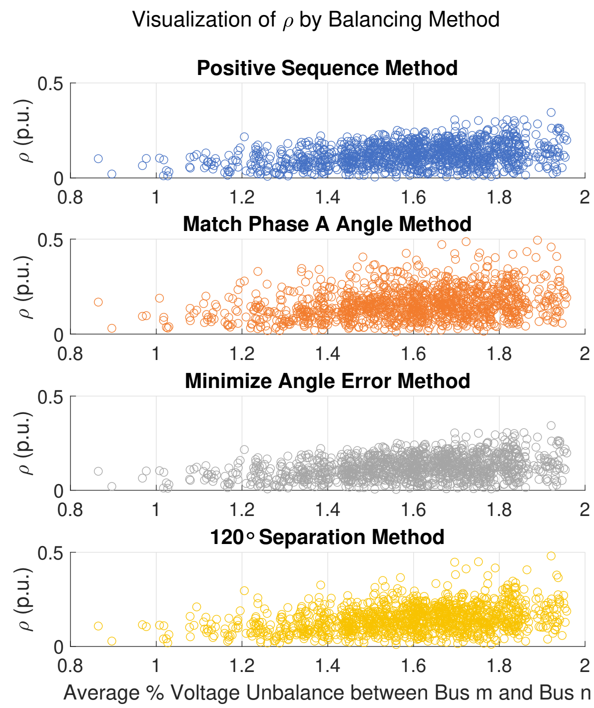

Fig. 5. Maximum absolute error in current flow across all three phases. This figure compares methods for computing balanced voltage phasors while abiding by the adapted IEC definition of voltage unbalance (4). Notice the significant spread from the Match Phase A Angle results compared to the concentrated results of the other three methods.

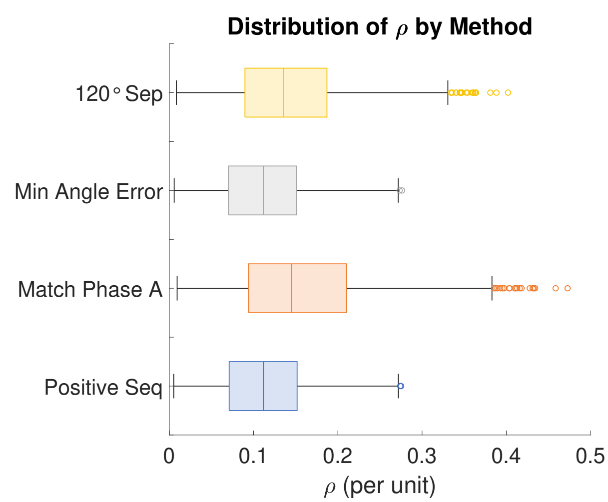

Fig. 6. The distribution of maximum absolute error in current flows compared between the four methods used to generate balanced voltage phasors.

peak loads in Table I are for the feeders themselves, and are adjusted by the scale factors before being passed to the transmission level simulation. The scale factors are chosen such that 80% of the peak feeder load corresponds to the nominal loading in the nine-bus system (nominal loads are shown in Fig. 7). This ensures that the transmission system remains stable as the individual feeder loads vary over time.

TABLE I

A SUMMARY OF METRICS FOR THE EPRI TEST CIRCUITS USED AS THE DISTRIBUTION FEEDERS FOR CO-SIMULATION.

Quantity Name	Power (units)	Ckt5	Ckt7	Ckt24
Phase A Unbalance	Real (%)	2.4	1.0	0.8
Phase B Unbalance	Real (%)	0.2	-1.4	-0.6
Phase C Unbalance	Real (%)	-2.6	0.5	-0.2
Peak Load	Real (MW)	19.2	67.9	50.0
Peak Load	Reactive (Mvar)	9.6	20.9	10.5
Scale Factor	Real	5.85	1.84	3.12
Scale Factor	Reactive	3.92	2.09	5.95

The hourly resolution load data available for these feeders permits each co-simulation to be run as a set of quasi-static simulations over an extended period of time, and therefore our results include a distribution of errors as opposed to the error at a single snapshot.

We characterize the errors associated with each balancing method by running an additional co-simulation that uses a full three-phase representation of the transmission system. This is considered the 'true' solution, and all errors are calculated by comparing to this result. The balanced transmission system power flows are solved using PyPower [24], while all three-phase distribution system power flows are solved in OpenDSS [23]. The co-simulation is managed using HELICS [25]. The current error, ρ, is computed in the same manner according to (14) for each branch in the system. The final reported value is averaged over the nine branches in the nine-bus system.

B. Results

In Fig. 8, the distributions of current errors in the nine-bus system are shown for a month-long quasi-static co-simulation for each of the four balancing methods. Recall that the current errors are calculated with respect to the co-simulation results using a three-phase representation of the feeders and the nine-bus system. The top pane shows ρ, the maximum error over the three phases, while the bottom panes show ρ_a, ρ_b, and ρ_c separately. The most obvious trend is that the Match A Angle Method performs significantly better for phase A, and significantly worse for phases B and C. This result is expected because this method matches the phase A angle exactly when generating the balanced representation. The 120 Degree Separation method performs better than the Match Phase A Angle Method but worse than the other two methods. Since the motivation for the 120 Degree Separation Method is to improve the two phases that are already close to balanced while sacrificing more error on the third phase, it is unsurprising that this method does not perform as well for a metric based on the maximum error of the three phases. The results for the co-simulation agree with the results from the two-bus system, thus providing validation using a larger test case.

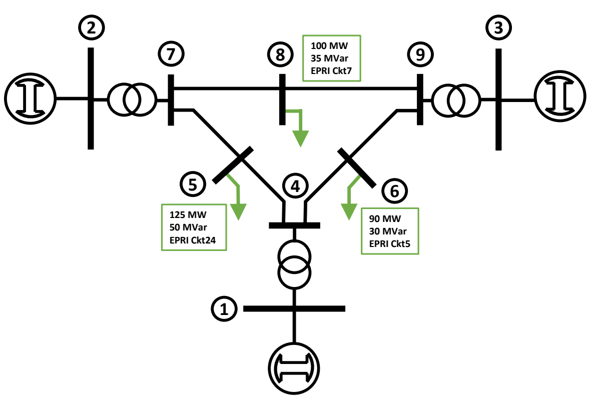

Fig. 7. The co-simulation system.

V. Conclusions and Future Work

This paper compares definitions for quantifying unbalanced conditions and methods for calculating a balanced representation by computing an error metric for the current flows in the system. We show results for both a two-bus test system and a transmission and distribution co-simulation consisting of the nine-bus WECC system and EPRI Distribution Test Feeders. The following are the main results and conclusions:

- Of the commonly used metrics for voltage unbalance, the IEC definition incurs the lowest average error in the current flows, regardless of the balancing method used.
- A potential shortcoming of the definition used in the IEC standard—that it does not consider the zero sequence component—is identified. We therefore also consider a more strict adapted IEC definition of unbalance that accounts for the zero sequence component in addition to the negative sequence component.
- For the two-bus system, the Positive Sequence Method and Minimize Angle Error Method perform better than

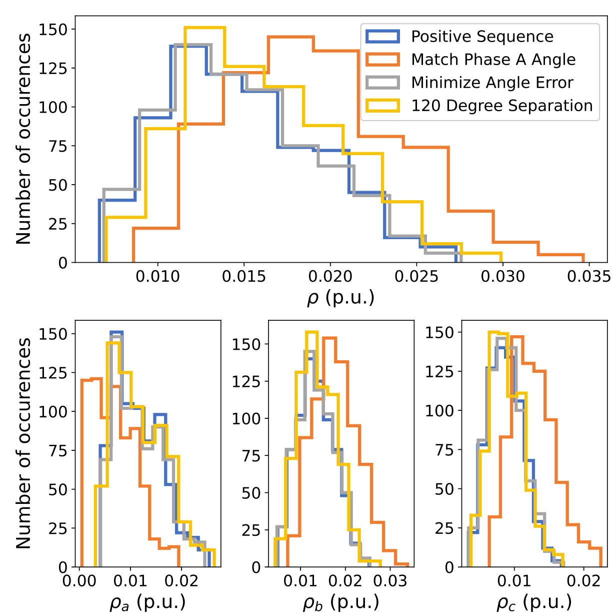

Fig. 8. Histogram of current flow errors across balancing methods for a one month, quasi-static co-simulation using the nine-bus WECC system.

the 120 Degree Separation Method and the Match Phase A Angle Method. This is true both on average and when considering the worst-case outliers.
- The results from the nine-bus co-simulation case study verify the results from the two-bus system. The Positive Sequence and Minimize Angle Error methods perform the best, while the Match Phase A Angle has the worst performance.

Future work in this area should strive to include analysis on even more realistic systems. This could include larger co-simulation systems with many transmission buses and distribution nodes in addition to systems corresponding to real datasets that augment the representative test cases considered in this paper. Given the possibility that sources of unbalance at the distribution level will cancel out to some degree when aggregated in large numbers, a system with many distinct distribution nodes would be an interesting extension to this work. Since the underlying goal of transmission and distribution co-simulation is to uncover complex interactions in large interconnected systems, understanding how the results shown here scale to much larger systems is key.

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