

# An Eigenvalue Formulation for Determining Initial Conditions of Induction Machines in Dynamic Power System Simulations

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**Abstract**—The problem of determining the initial conditions for induction machine variables in a non-linear dynamic power system analysis is posed as an eigenvalue problem. The eigenvalue formulation can then be solved using standard linear algebra techniques. This method has the advantage of determining all possible solutions for the internal variables rather than the single solution obtained from traditional iterative methods. Additionally, the method can easily determine when the problem has no solutions through the absence of non-zero real eigenvalues.

## I. INTRODUCTION

Dynamic simulations of power systems generally begin by running a load flow analysis to determine steady state values of the real power ( $P$ ), the reactive power ( $Q$ ), the voltage magnitude ( $V$ ) and the voltage angle ( $\delta$ ) at each bus. Subsequently, the initial conditions for components connected to the bus are set to their steady state value. Induction machines comprise a significant portion of the load at many buses. For each induction machine connected to a bus, the initial conditions of the machine model's internal state variables must be obtained such that the terminal variables of the model match the values of  $P$ ,  $Q$ ,  $V$ , and  $\delta$  obtained from the load flow analysis. Appropriate choice of the internal states of the induction machine model can set both the machine's real power and voltage to match the bus values that are obtained from the load flow analysis. An additional capacitor is often added at the bus to ensure that the reactive power output of the machine matches the reactive power obtained from the load flow analysis [1].

A common approach for matching the real power and voltage uses iteration on the induction machine's slip to solve the machine equations for the internal variables using the known terminal values. This approach is used in such software as Electro-Magnetic Transients Program (EMTP) [2, pp. 9-19], Positive Sequence Load Flow Software (PSLF) [3, pp. 785] and Power System Simulation for Engineering (PSS/E) [4, pp. 20-16]. A potential disadvantage of the iteration approach is that only one solution is obtained when multiple solutions are often possible. For general purposes the highest speed stable solution is sought; however, for some research purposes, studies can focus on other unstable solutions.

In this paper, we present an eigenvalue approach for solving the induction machine initial conditions problem. Assuming linear magnetic relationships, the electrical equations appear nearly linear, with non-linearity involving a common multiplier, the rotor speed. This will serve as the eigenvalue in our formulation. The eigenvector is composed of the machine electrical variables. Once these are solved, the torque equation is used to initialize the model's mechanical torque.

This eigenvalue approach has the advantage of providing all solutions, stable and unstable, and can reliably determine when no solution exists through the absence of non-zero real eigenvalues.

## II. DYNAMIC INDUCTION MACHINE MODEL

This section presents a standard dynamic induction machine model in the dq frame with linear magnetic relationships and short circuited rotor windings. The model was adapted from [5, pp. 284].

$$\begin{aligned} \begin{bmatrix} V_{ds} \\ V_{qs} \\ 0 \\ 0 \end{bmatrix} &= \begin{pmatrix} \begin{bmatrix} R_s & -(X_{ls} + X_m) & 0 & -X_m \\ (X_{ls} + X_m) & R_s & X_m & 0 \\ 0 & -X_m & R_r & -(X_{lr} + X_m) \\ X_m & 0 & (X_{lr} + X_m) & R_r \end{bmatrix} \\ + \frac{\omega_r}{\omega_s} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & X_m & 0 & (X_{lr} + X_m) \\ -X_m & 0 & -(X_{lr} + X_m) & 0 \end{bmatrix} \end{pmatrix} \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_{dr} \\ I_{qr} \end{bmatrix} \\ + \frac{1}{\omega_s} \begin{bmatrix} (X_{ls} + X_m) & 0 & X_m & 0 \\ 0 & (X_{ls} + X_m) & 0 & X_m \\ X_m & 0 & (X_{lr} + X_m) & 0 \\ 0 & X_m & 0 & (X_{lr} + X_m) \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_{dr} \\ I_{qr} \end{bmatrix} \end{aligned} \quad (1)$$

$$\frac{2H}{\omega_s} \frac{d\omega_r}{dt} = T_e(I_{ds}, I_{qs}, I_{dr}, I_{qr}) - T_m \quad (2)$$

$$T_e(I_{ds}, I_{qs}, I_{dr}, I_{qr}) = X_m (I_{qs} I_{dr} - I_{ds} I_{qr}) \quad (3)$$

$\omega_s$  refers to the electrical excitation frequency and  $\omega_r$  refers to the rotor speed.  $X_{ls}$  is the stator leakage reactance,  $X_{lr}$  is

the rotor leakage reactance,  $X_m$  is the mutual reactance,  $R_s$  is the stator resistance,  $R_r$  is the rotor resistance, and  $H$  is the inertia constant of the machine and mechanical load. All quantities are in per unit.

The familiar steady state equivalent circuit for the induction machine model from (1), (2), and (3) is as follows. The machine slip is  $s = \frac{\omega_s - \omega_r}{\omega_s}$ , the stator current is  $I_s = I_{ds} + jI_{qs}$ , the rotor current is  $I_r = I_{dr} + jI_{qr}$ , and the stator voltage is  $V_s = V_{ds} + jV_{qs}$ .

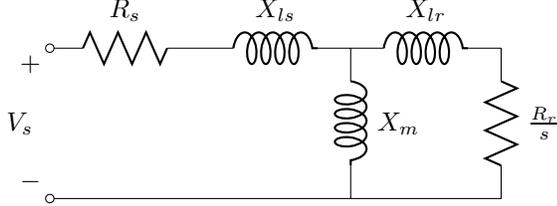


Fig. 1. Induction Machine Steady State Equivalent Circuit

### III. EIGENVALUE FORMULATION OF THE INITIAL CONDITIONS PROBLEM

The dynamic induction machine model given by (1), (2), and (3) fits into a more general induction machine dynamic model framework.

$$y = [\mathbf{A} + \omega_r \mathbf{B}] x + \mathbf{C} \frac{dx}{dt} \quad (4)$$

$$\frac{2H}{\omega_s} \frac{d\omega_r}{dt} = T_e(x) - T_m \quad (5)$$

This model has the applied voltage contained in the vector  $y$  and stator and rotor currents contained in the vector  $x$ . The rotor windings are short circuited.  $\mathbf{A}$  contains all terms that do not depend on the rotational speed  $\omega_r$ ,  $\mathbf{B}$  contains all terms that do depend on  $\omega_r$ , and  $\mathbf{C}$  contains all terms that depend on the derivative of the currents. The model thus has a single multiplicative non-linearity, namely a dependence on  $\omega_r$ .

In steady state,  $\frac{d\omega_r}{dt} = 0$  and  $\frac{dx}{dt} = 0$ .

$$y = [\mathbf{A} + \omega_r \mathbf{B}] x \quad (6)$$

$$T_e(x) = T_m \quad (7)$$

$\mathbf{A}$  and  $\mathbf{B}$  are completely defined by the machine parameters. Since the phase angle of the applied voltage does not affect the power consumption of the motor, we specify the stator d-axis voltage  $V_{ds}$  equal to the voltage obtained from the load flow analysis and the stator q-axis voltage  $V_{qs}$  equal to zero. The voltage is then directed entirely in the d-axis. This specification can be corrected at the end of the method by rotating the current angles by the bus angle obtained from the load flow analysis. Therefore, the voltage vector  $y$  is completely known. Since  $V_{qs}$  is specified to be zero, the real power used by the machine is  $P = V_{ds} I_{ds}$ . The real power is known from the load flow analysis, so the d-axis current can be directly determined.

$$I_{ds} = \frac{P}{V_{ds}} \quad (8)$$

The machine model can be put into the form of an eigenvalue problem by combining the known voltage vector and the known matrix  $\mathbf{A}$ . First rewrite the voltage vector  $y$  as

$$y = \begin{bmatrix} V_{ds} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{V_{ds}}{I_{ds}} \\ 0 \\ 0 \\ 0 \end{bmatrix} I_{ds} = \begin{bmatrix} R_A \\ 0 \\ 0 \\ 0 \end{bmatrix} I_{ds} \quad (9)$$

where, using (8),

$$R_A = \frac{V_{ds}}{I_{ds}} = \frac{V_{in}^2}{P} \quad (10)$$

Then define the matrix  $\mathbf{D}$  as

$$-\mathbf{D} = - \left[ \begin{array}{c|c} R_A & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} \end{array} \right] + \mathbf{A} \quad (11)$$

and rewrite (6) as

$$-\mathbf{D}x + \omega_r \mathbf{B}x = 0 \quad (12)$$

or, in a well-known generalized eigenvalue form

$$\mathbf{D}x = \omega_r \mathbf{B}x \quad (13)$$

This formulation can either be solved with generalized eigenvalue solution techniques or, since  $\mathbf{D}$  is generally invertible under the condition that stator and rotor resistances are specified, converted to standard eigenvalue form and solved with standard eigenvalue techniques [6].

$$\frac{1}{\omega_r} x = \mathbf{D}^{-1} \mathbf{B}x \quad (14)$$

Since the eigenvector can be arbitrarily scaled, rescaling the eigenvector using the known value of  $I_{ds}$  from (8) is required. Additionally, correction for the voltage angle  $\delta$  from the load flow analysis is needed: rotate the current vector  $x$  by  $\delta$  after solving the eigenvalue problem. Finally, the mechanical torque is obtained using (3) and (7).

### IV. NUMERIC EXAMPLE

Assume  $V_{in}$  is obtained from a load flow analysis. Specify  $V_{ds} = V_{in}$  and  $V_{qs} = 0$ . The method can then be applied to the induction machine model given by (6) and (7) as previously described in section III.

The matrix  $\mathbf{D}$  is

$$\mathbf{D} = - \begin{bmatrix} (R_s - R_A) - (X_{ls} + X_m) & 0 & -X_m \\ (X_{ls} + X_m) & R_s & X_m & 0 \\ 0 & -X_m & R_r & -(X_{lr} + X_m) \\ X_m & 0 & (X_{lr} + X_m) & R_r \end{bmatrix} \quad (15)$$

Now  $\omega_r$  and the currents in  $x$  can then be found by solving (14). Note that the eigenvector  $x$  must be scaled correctly. Since the first entry of  $x$  should be  $I_{ds}$ , scaling can be done by multiplying the each entry in the eigenvector  $x$  by  $I_{ds}$  divided by the first entry of  $x$ .

Consider an induction machine with the following parameter values in per unit representation.

$X_{ls}$	$X_{lr}$	$X_m$	$R_s$	$R_r$	$\omega_s$ (rad/sec)
0.10	0.10	3.5	0.013	0.015	377

TABLE I  
INDUCTION MACHINE EQUIVALENT CIRCUIT PARAMETERS

Assume that the following values are obtained from a load flow analysis.

$P$	$V_{in}$	$\delta$
1.0	1.0	$30^\circ$

TABLE II  
LOAD FLOW PARAMETERS

From this data, (8) shows that  $I_{ds} = 1.0$  and (10) shows that  $R_A = 1.0$ .  $\mathbf{D}$  can then be obtained from (15).

$$\mathbf{D} = \begin{bmatrix} 0.987 & 3.6 & 0 & 3.5 \\ -3.6 & -0.013 & -3.5 & 0 \\ 0 & 3.5 & -0.015 & 3.6 \\ -3.5 & 0 & -3.6 & -0.015 \end{bmatrix}$$

Solving (14) gives two non-zero solutions for  $\omega_r$ . The zero eigenvalues can be neglected since they are not physically meaningful. The eigenvector  $x$  was scaled such that the  $I_{ds}$  entry is equal to its known value of 1.0. After scaling the eigenvector, the currents were rotated by  $\delta$ . This was accomplished by calculating the vectors  $I_k = (I_{dk} + jI_{qk})e^{j\delta}$  for  $k = \{s, r\}$ . The rotated stator and rotor d-axis currents correspond to the real parts of  $I_k$ , and the rotated stator and rotor q-axis currents correspond to the imaginary parts of  $I_k$ . Finally, torque was calculated using (3).

The solutions for  $\omega_r$  in radians per second and the currents and torque in per unit are presented below. Note that both the stable and unstable solutions are acquired rather than the single solution that would be obtained from an iterative method. These solutions were verified in the steady state equivalent circuit shown in Fig. 1 through the traditional method of iteration on the machine's slip until the proper terminal power was obtained.

Solution	$\omega_r$ ( $\frac{\text{rad}}{\text{sec}}$ )	$I_{ds}$	$I_{qs}$	$I_{dr}$	$I_{qr}$	$T_m$
1	183.6	3.295	-3.708	-3.233	3.579	0.6801
2	370.7	1.110	0.0773	-0.999	-0.323	0.9839

TABLE III  
SOLUTION TO INDUCTION MACHINE EXAMPLE

The torque versus speed curve for this machine is given in Fig. 2, and both solutions are shown.

## V. CONCLUSION

A new method for determining the initial conditions of induction machine models in dynamic power system studies has been developed. This method converts the initial conditions problem into a generalized eigenvalue formulation that

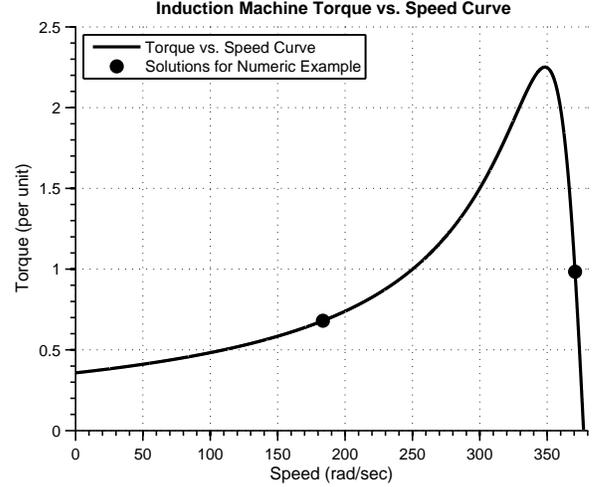


Fig. 2. Torque Speed Curve and Solutions for Numeric Example

can be solved using standard linear algebra techniques. In contrast to existing iterative methods, the eigenvalue method has the advantage of providing all solutions to the initial conditions problem and can reliably determine when no solutions exist. While eigenvalue solution techniques for arbitrarily sized matrices require iteration, these techniques are robust. Additionally,  $\mathbf{D}^{-1}\mathbf{B}$  in (14) has rank 2. Therefore, the largest factor in the eigenvalue characteristic equation for the induction machine model is a second order polynomial. Hence, the eigenvalue formulation can be non-iteratively solved using the quadratic formula.

Additional work on eigenvalue formulations for induction machine problems is promising. For instance, the method presented in this paper is directly applicable to other induction machine models, such as a double-cage induction machine model. The method could also be extended to induction machines with non-linear magnetic relationships in order to incorporate saturation effects. This could be accomplished by solving the eigenvalue formulation inside an iteration process.

## ACKNOWLEDGMENT

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