Counterexample to a Continuation-Based Algorithm for Finding All Power Flow Solutions

Daniel K. Molzahn†, *Student Member, IEEE*, Bernard C. Lesieutre†, *Senior Member, IEEE*, and Heng Chen‡

***Abstract*—Existing literature claims that an algorithm based on continuation is capable of finding all solutions to the power flow equations for all power systems. This claim is demonstrated to be incorrect through the use of a five-bus system counterexample. Existing literature also claims that a similar algorithm is capable of finding all Type-1 solutions (i.e., solutions where the power flow Jacobian has a single eigenvalue with positive real part) to the power flow equations for all systems. This claim is also shown to be incorrect using the five-bus system counterexample.**

***Index Terms*—Continuation power flow, voltage stability**

I. Introduction

THE power flow equations model the relationship between voltages and active and reactive power injections in a power system. It is well known that large numbers of solutions to these equations can exist. Power systems are typically operated at the high-voltage, stable solution, for which numerous solution techniques have been developed (e.g., Newton-Raphson, Gauss-Seidel, etc.). However, other solutions are also of interest.

A direct approach to finding multiple power flow solutions simply initializes Newton-Raphson iterations over a range of carefully selected candidate initial conditions. However, this approach does not guarantee obtaining all power flow solutions. In another approach, Salam *et al.* [1] apply the homotophy method of Chow *et al.* [2] to the power flow problem. This method can reliably find all solutions, but has a computational complexity that grows exponentially with system size. It is not computationally tractable for large systems.

Ma and Thorp published a continuation-based algorithm that they claimed would reliably find all solutions to the power flow equations [3], [4]. Since the computational complexity of this algorithm scales with the number of actual, rather than possible, solutions, it is computationally tractable for large systems. In other publications, a similar algorithm is used to find all Type-1 power flow solutions [5]. Type-1 solutions are those where the Jacobian of the power flow equations has a single eigenvalue with positive real part. Type-1 solutions are closely related to voltage instability phenomena [6].

In this letter, we present a five-bus system counterexample to the claim that the continuation-based algorithm will reliably find all solutions to the power flow equations. In the related thesis [7], a flaw in the proof associated with the completeness of the continuation-based algorithm is presented. The ten solutions to the five-bus system were calculated using a homotopy method [1]. There are three groups of solutions that, while connected by continuation traces to all other solutions within the group, are not connected to solutions outside of the group. Thus, the continuation-based algorithm fails to find all solutions. Furthermore, since a Type-1 solution exists for this system, the five-bus system also provides an example where the continuation-based algorithm fails to find all Type-1 solutions.

University of Wisconsin-Madison Department of Electrical and Computer Engineering:† molzahn@wisc.edu, lesieutre@wisc.edu; Electric Power Group, LLC.:‡ chen@electricpowergroup.com

II. Overview of the Continuation-Based Algorithm

The continuation-based algorithm [3], [4] modifies the power flow equations by adding a scalar parameter α to the active or reactive power equation for a bus. The algorithm starts from a single power flow solution obtained using traditional methods. At each step in the algorithm, the modified power flow equations are solved after changing α by a small amount. This creates a continuation trace. Solutions to the power flow equations are obtained when $\alpha = 0$. The continuation trace terminates when the trace returns to its starting point. Existing literature [3], [4] claims that all solutions are connected by these continuation traces. Thus, if continuation traces are started from each solution for each parameter (i.e., each solution/parameter pair) all solutions will be obtained; at most $\frac{ns}{2}$ continuation traces are required to find all solutions, where n is the number of buses and s is the number of solutions.

III. Five-Bus System Counterexample

The five-bus system given in Fig. 1 provides a counterexample to the claim that the continuation-based algorithm finds all solutions to the power flow equations for all power systems. Line values are given in per unit impedance and power injections are given in MW. We use a 100 MVA base.

We expect that the algorithm may fail to find all solutions for systems that have non-radial, weakly connected regions that have strong voltage support. In this example, bus three is weakly connected (i.e., connected via high impedance lines) to the rest of the network. Since they consist of PV buses, both bus three and the rest of the network (including the slack bus at bus one) have strong voltage support.

The ten solutions to the power flow equations for the five-bus system are given in Table I. Since the system contains only slack and PV buses, the voltage magnitude is specified at each bus (all voltage magnitudes are 1.000 per unit). δ_i is the voltage angle at bus i in degrees.

The continuation traces for this system using active power parameters and starting from solution one are shown in Fig. 2. These continuation traces only contain solutions one and two. The continuation traces started from solution two are identical to those in Fig. 2 and also only contain solutions one

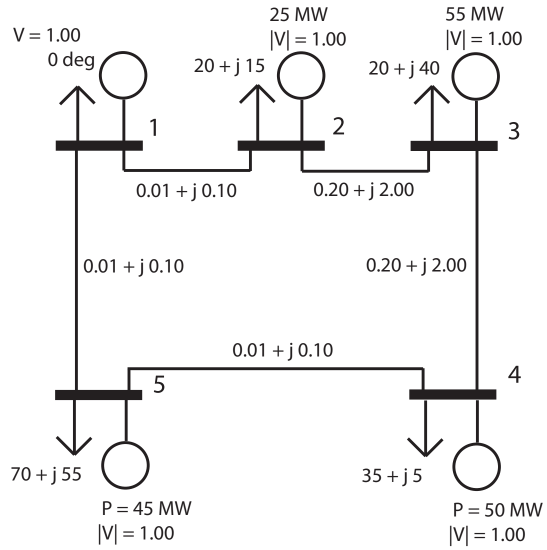

Fig. 1. Five-Bus System

Solution	δ_1	δ_2	δ_3	δ_4	δ_5
1	0	1.286	22.061	2.194	0.372
2	0	0.166	171.198	0.028	-0.710
3	0	-169.906	-148.192	-167.129	-168.909
4	0	-168.702	3.182	-167.131	-167.912
5	0	2.187	45.923	46.616	-143.973
6	0	-168.657	-172.863	44.012	-145.341
7	0	-171.391	-99.227	50.716	-141.807
8	0	-0.897	-168.405	44.388	-145.144
9	0	-169.370	-10.988	165.903	-25.378
10	0	-169.282	-160.897	166.147	-22.898

TABLE I
ALL SOLUTIONS TO THE FIVE-BUS SYSTEM

and two. These continuation traces do not find solutions three though ten. Thus, solutions one and two are disconnected from the eight other solutions. Similarly, continuation traces started from solutions three and four only find solutions three and four, and continuation traces started from any of the remaining solutions five through ten only find solutions five through ten (plots of the other continuation traces are excluded for brevity). The continuation-based algorithm therefore fails to find all solutions.

The eigenvalues of the power flow Jacobian were evaluated at each solution. With a single eigenvalue that has positive real part, solution two is the only Type-1 solution. This solution cannot be reached from continuation traces that start from solutions three through ten. Thus, the five-bus system also provides a counterexample to the claim in [5] that the continuation-based algorithm can reliably find all Type-1 solutions.

IV. CONCLUSION

This letter has presented a five-bus system counterexample to the claim in existing literature that the continuation-based algorithm is capable of finding all solutions to the power flow equations for all systems. Since other methods for finding all

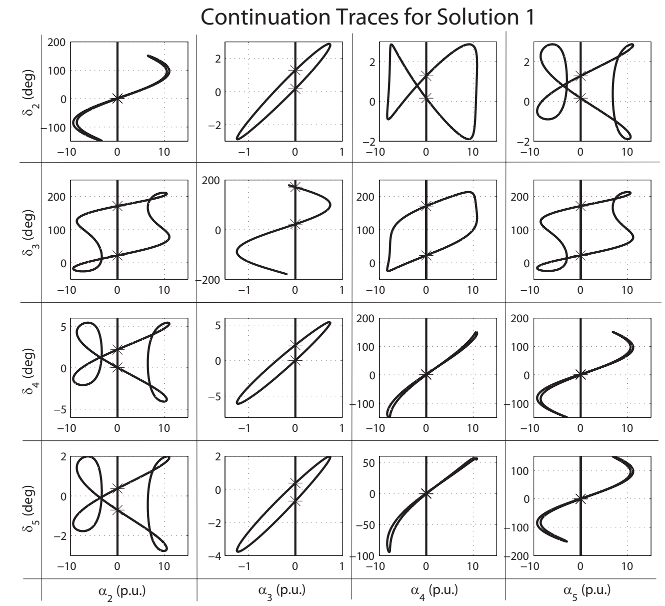

Fig. 2. Continuation Traces for Solution 1

solutions to the power flow equations are not computationally tractable for large systems, we conclude that there is presently no method for reliably computing all solutions to the power flow equations for practically sized systems.

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