Identifying Redundant Constraints for AC OPF: The Challenges of Local Solutions, Relaxation Tightness, and Approximation Inaccuracy

Alejandro D. Owen Aquino,[1] Line A. Roald,[2] and Daniel K. Molzahn[1]

***Abstract*— To compute reliable and low-cost operating points, electric power system operators solve optimization problems that enforce inequality constraints such as limits on line flows, voltage magnitudes, and generator outputs. A common empirical observation regarding these constraints is that only a small fraction of them are binding (satisfied with equality) during operation. Furthermore, the same constraints tend to be binding across time periods. Recent research efforts have developed constraint screening algorithms that formalize this observation and allow for screening across operational conditions that are representative of longer time periods. These algorithms identify redundant constraints, i.e., constraints that can never be violated if other constraints are satisfied, by solving optimization problems for each constraint separately. This paper investigates how the choice of power flow formulation, represented either by the non-convex AC power flow, convex relaxations, or a linear DC approximation, impacts the results and the computational time of the screening method. This allows us to characterize the conservativeness of convex relaxations in constraint screening and assess the efficacy of the DC approximation in this context.**

I. Introduction

Optimization problems are commonly used in the day-to-day operation, protection, and expansion of the electric grid. Problems such as the optimal power flow (OPF) [1] provide engineers and grid operators valuable insights that inform their planning and control decisions. These optimization problems usually incorporate both physical constraints to model the power flow equations and engineering constraints such as voltage, generator, angle difference, and line flow limits. As a result, the number of variables and constraints increase rapidly with the network size, leading to computational challenges for large systems.

While all the constraints in these optimization problems must be satisfied for a solution to be feasible, previous work has shown many constraints are never binding (i.e., satisfied with equality) at the optimal solution [2]. This opens the possibility for screening out non-binding constraints to reduce the problem size, speed up computations, and improve numerical conditioning. Additionally, analyzing the binding constraints provides insights into system operations by identifying which components are operated at their limits.

Constraint screening algorithms have been proposed for DC OPF and DC unit commitment in [3]–[6] and applied in the context of load variability in [7]–[9]. These approaches solve an optimization problem associated with each line flow limit to identify and remove the limits that never become binding across a large range of load fluctuations that is representative of long-term load variability. The existing work on AC constraint screening, on the other hand, comes from the bound tightening literature, where the objective is to tighten bounds on voltage magnitude and angle difference limits to improve the quality of convex relaxations [10]. This leads to some differences in the approach and large differences in motivation and interpretation relative to constraint screening.

This paper's main contribution is a method for identifying redundant constraints in AC OPF problems for a range of operational conditions by adapting an existing bound tightening method. The proposed method is ultimately intended as a prepossessing step that can be infrequently computed offline to obtain a simplified mathematical model that is useful for a range of subsequent optimization problems. We use numerical experiments to first determine the effectiveness of this method in identifying constraints that are found to be redundant for some operating point and then evaluate the impact of removing these constraints on AC OPF computational times. Lastly, we apply the proposed method with two different convex relaxations and the DC approximation to compare the relative effectiveness of these formulations.

The reminder of the paper is organized as follows. Section II describes the AC OPF problem. Section III presents a methodology to identify both the redundant and the binding constraints in the AC OPF problem with load variability. Section IV discusses the numerical results for several test cases, followed by conclusions in Section V.

II. AC Optimal Power Flow Problem Formulation

The AC OPF problem computes the least-cost operating point subject to physical laws governing power flows and engineering constraints on line flows, power generation, and voltages. A power system consists of a set $\mathcal{N}$ of buses (a subset of which have generators denoted by the set $\mathcal{G}$), and a set $\mathcal{L}$ of lines. There are a variety of different OPF formulations [11], and here we use the following:

$$\min_{V,S_G} \sum_{k\in\mathcal{G}} c_{k_2}P_{G_k}^2 + c_{k_1}P_{G_k} + c_{k_0} \tag{1a}$$

$$\text{s.t.}\quad (V_k^{\min})^2 \le V_k V_k^* \le (V_k^{\max})^2 \qquad \forall k\in\mathcal{N} \tag{1b}$$

$$P_k^{\min} \le P_{G_k} \le P_k^{\max} \qquad \forall k\in\mathcal{G} \tag{1c}$$

$$Q_k^{\min} \le Q_{G_k} \le Q_k^{\max} \qquad \forall k\in\mathcal{G} \tag{1d}$$

$$I_{ij}I_{ij}^* \le (I_{ij}^{\max})^2 \qquad \forall (i,j)\in\mathcal{L} \tag{1e}$$

$$-\theta_{ij}^{\max} \le \theta_{ij} \le \theta_{ij}^{\max} \qquad \forall (i,j)\in\mathcal{L} \tag{1f}$$

$$I^f = Y_f V \tag{1g}$$

[1]: School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA. {aaquino,molzahn}@gatech.edu. Support from the National Science Foundation Energy, Power, Control and Networks under award 2023140.

[2]: Department of Electrical and Computer Engineering, University of Wisconsin–Madison, Madison, WI. roald@wisc.edu.

$$I^t = Y_t V \quad (1h)$$
$$\theta_{ij} = \angle(V_i V_j^*) \qquad \forall (i,j) \in \mathcal{L} \quad (1i)$$
$$\theta^{ref} = 0 \quad (1j)$$
$$P_{G_k} + jQ_{G_k} = S_{G_k} \qquad \forall k \in \mathcal{G} \quad (1k)$$
$$S_{G_k} - S_{D_k} = \sum_{(k,j)\in\mathcal{L}} S_{kj} \qquad \forall k \in \mathcal{N} \quad (1l)$$
$$S_{ij} = Y_{ij}^* V_i V_i^* - Y_{ij}^* V_i V_j^* \qquad \forall (i,j) \in \mathcal{L} \quad (1m)$$

The objective (1a) minimizes the generation cost, with c_{k_2}, c_{k_1} and c_{k_0} denoting the coefficients for a quadratic cost function. The voltage magnitudes at each bus $k \in \mathcal{N}$ are bounded by upper and lower limits $V_k^{\min}$ and $V_k^{\max}$ in (1b), which is described as a quadratic constraint. The superscript * denotes the complex conjugate. Similarly, the active and reactive power outputs of each generator $k \in \mathcal{G}$ are bounded by $P_k^{\min}$, $P_k^{\max}$, $Q_k^{\min}$ and $Q_k^{\max}$ in (1c) and (1d), respectively. The current flows I_{ij} at each line $(i,j) \in \mathcal{L}$ are defined in (1g)–(1h) where I^t and I^f are vectors containing the current flow phasors for the "to" (t) and "from" (f) directions of the branches, V is the vector of voltage phasors, and Y_f and Y_t are the system branch admittance matrices corresponding to the current flows into the "from" (f) and "to" (t) ends of the lines. The squared magnitudes of the current flows are upper bounded by $(I_{ij}^{\max})^2$ in (1e). The angle differences between buses connected by lines $(i,j) \in \mathcal{L}$ are defined in (1i) and bounded by $\theta_{ij}^{\max}$ in (1f). The voltage angle at the reference bus is set to zero in (1j) The complex power flow S_{ij} is defined in (1m). Lastly, power balance is enforced in (1l), where S_{D_k} is the power demand at each bus. Y refers to the bus admittance matrix of the network. The inequality constraints in (1b)–(1f) are the focus of this paper.

The non-linear power flow equations (1m) induce non-convexities that can lead to locally (as opposed to globally) optimal solutions. Convex relaxations of these equations mitigate this difficulty by bounding the optimal objective values of nonconvex AC OPF problems. In the context of constraint screening, this is particularly important since locally optimal solutions can lead to a constraint being wrongfully characterized as redundant (and thus omitted from the problem), as explained later in this paper. Two commonly used AC power flow relaxations are the quadratic convex (QC) relaxation and the semidefinite programming (SDP) relaxation. The SDP relaxation [12] represents (1) as a rank-constrained problem and then drops the rank constraint to form a relaxation. The QC relaxation [13] constructs convex envelopes of the polar representation of the power flow equations. We will also consider the computationally efficient DC power flow approximation [14], which ignores reactive power and voltage magnitudes. For brevity, formulations of the SDP relaxation, QC relaxation, and DC approximation of the AC OPF problem are omitted in this paper.

III. AC Constraint Screening Methodology

The screening method proposed in this paper uses optimization problems to minimize or maximize various quantities subject to all other constraints in the AC OPF problem. If the maximum and minimum achievable values (subject to all other constraints) for a quantity are within its specified limits, then those limits are *redundant* and can be eliminated from the problem. In other words, enforcing the combination of certain constraints can imply the satisfaction of other constraints. If the quantity in question reaches its established limits, however, there exists some feasible operating point for which that constraint becomes binding. In this case, we consider the constraint to be *non-redundant*. We note that this method only considers whether a constraint could potentially be binding given the other constraints in the problem, not whether typical generation cost functions would result in this constraint being binding. Each voltage limit, angle difference limit, line current flow limit, and generator active and reactive power limit can be treated with this screening procedure.

Redundant constraint: A constraint that will never become binding (satisfied with equality) if all other constraints are satisfied. Since the satisfaction of other constraints implies that this constraint it satisfied, it can be safely neglected. **Non-redundant constraint:** There is at least one feasible operating point in the considered operating range where this constraint becomes binding. As a result, this constraint cannot be safely neglected.

Inconclusive constraint: A constraint that is not classified as either redundant or non-redundant.

Since we are interested in screening out constraints that are redundant for a large set of possible operating points, we solve our maximization and minimization problems for each constraint over a range of load variability. Every active load P_{D_k} and reactive load Q_{D_k} for $k \in \mathcal{D}$ then become extra decision variables in the optimization problem. This gives rise to the optimization problem shown below, where the objective function (2a) minimizes/maximizes one of the quantities from constraints (1b)–(1f), subject to all other AC OPF constraints (1k)–(1m) and the load variability constraints (2b)–(2c). The parameter δ controls how much loads are allowed to vary, and $P_{D_k}^o$ and $Q_{D_k}^o$ refer to the nominal loads given by the test case.

$$\min_{V,S_G,S_D} / \max_{V,S_G,S_D} \; V_k/P_n/Q_n/I_{ij}^2/\theta_{ij}, \quad \forall k \in \mathcal{N}, \forall n \in \mathcal{G}, \forall (i,j) \in \mathcal{L} \quad (2a)$$
$$\text{s.t.} \quad (1-\delta)P_{D_k}^o \le P_{D_k} \le (1-\delta)P_{D_k}^o, \quad \forall k \in \mathcal{D} \quad (2b)$$
$$(1-\delta)Q_{D_k}^o \le Q_{D_k} \le (1-\delta)Q_{D_k}^o, \quad \forall k \in \mathcal{D} \quad (2c)$$
$$(1b) - (1m) \quad (2d)$$

A. Addressing nonconvexities via convex relaxations

To rigorously classify a constraint as redundant, we need to find the most extreme achievable value of the constrained quantity over the operating range and verify that it is within its specified limits. This is only possible by considering the globally optimal objective values for (2). Globally solving (2) can be challenging, as OPF problems are NP-hard in general [15]. Therefore, instead of trying to solve the nonconvex problem, we use a convex relaxation to bound the global solution. If the bound from a relaxation is within the specified limits for the constrained quantity, then we are assured that the constraint is indeed redundant. We will compare the performance of the sparsity-exploiting SDP relaxation [16] and the QC relaxation with bound tightening [17], both formulated with current flow limits.

An additional benefit of using the QC relaxation is that we can use iterative bound tightening strategies on voltage magnitudes and angle differences to help with the constraint screening in two ways:

1) Bound tightening methods can improve the QC relaxation considerably, reducing the optimality gap to within 1% for many cases [10]. Tighter relaxations improve the ability of the constraint screening method to characterize constraints as redundant.
2) As a byproduct of tightening the QC relaxation, bound tightening inherently identifies redundant voltage and angle difference constraints, eliminating the need to solve extra optimization problems for those constraints.

Fig. 1 shows how the constraint screening method uses SDP and QC relaxations of (2) to identify redundant constraints.

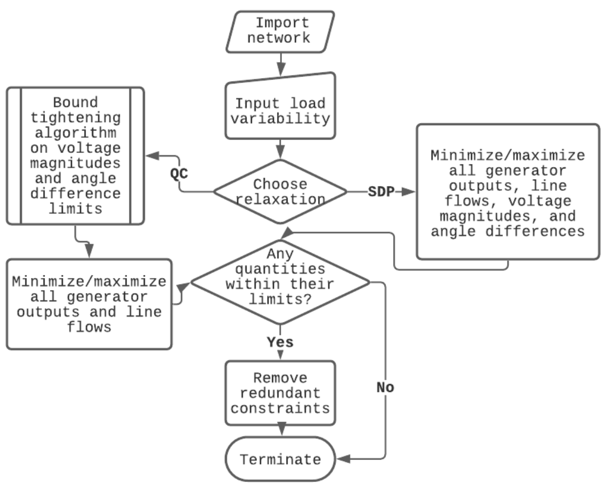

Fig. 1. AC constraint screening flow chart.

B. *Finding non-redundant constraints*

Convex relaxations can be used to bound the objective value of a constraint screening problem (2). If this bound is within the specified limit, then the constraint will never be binding. However, relaxations do not guarantee that a constraint can become binding. In other words, a relaxation can find those constraints which are definitely redundant, but not those which are definitely non-redundant. This comes from the fact that there may still exist a relaxation gap even after applying a bound tightening algorithm.

For this reason, we solve the maximization and minimization problems with the full nonconvex AC power flow equations to find non-redundant constraints. If we find that a constrained quantity can reach or violate its limits, we can guarantee that the constraint will become binding for some point within the operating region (even if we did not find the point with the worst-case violation).

Fig. 2 illustrates how the convex relaxation and the nonconvex problem are used to find which constraints are redundant and non-redundant, respectively. We note that there

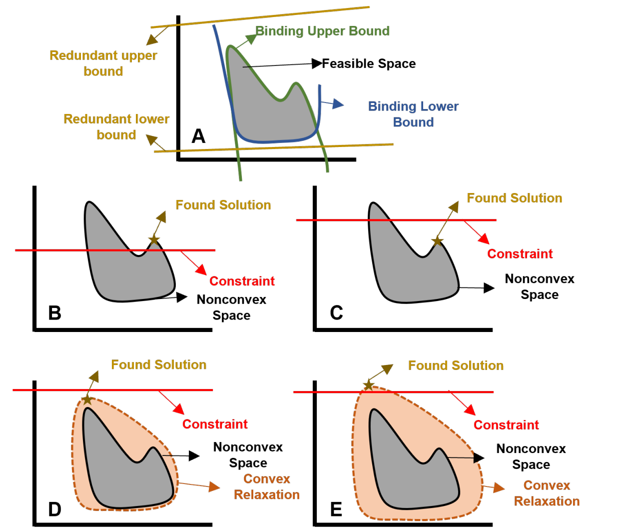

Fig. 2. In this illustration, a grey feasible region is defined in figure A by the green upper bound and blue lower bound. Figure A also shows redundant upper and lower bounds. In figures B through E, the constraint shown by the red line is another upper bound that is being evaluated for redundancy. The feasible region of a relaxation is shown in orange. In figure B, the solution to the nonconvex problem is used to guarantee that a constraint is non-redundant since it found an operating point that would violate the constraint even though it only found a local optimum. In figure C, the same nonconvex problem fails to find an operating point that would violate the constraint because it could only solve to local optimality, thereby falsely labeling the constraint as redundant. In figure D, the solution to the convex relaxation is used to guarantee that the constraint is redundant by showing that the upper bound on the maximum value does not violate the constraint. Lastly, in figure E, a loose convex relaxation finds a possible constraint violation whereas the actual constraint cannot be violated, thereby falsely labeling the constraint as non-redundant. With our method, the situations in figures C and E result in the constraint being labeled as inconclusive.

are cases where this approach will yield inconclusive results for some constraints. This happens when the solution to the relaxed problem suggests a possible constraint violation (and thus the constraint is potentially non-redundant) but the solution obtained for the nonconvex problem is within the limit (and thus the constraint is potentially redundant). In this case, the constraint is characterized as inconclusive.

The above discussion applies to the convex relaxations. When using the DC approximation, we only label constraints as redundant or non-redundant *for the DC OPF problem* which uses the DC power flow approximation. We note that a constraint that is classified as redundant or non-redundant using the DC formulation is *not* necessarily redundant or non-redundant for the AC OPF. The results in the following section thus use the DC approximation for the sake of comparison to previously proposed constraint screening methods.

IV. Results

This section demonstrates the methods described in Section III for various PGLib-OPF test cases [18]. Modified versions of the software packages PowerModels.jl and PowerModelsAnnex.jl [19] were used for the implementation in Julia. All non-linear programs (NLPs) were solved using Ipopt [20], while all SDPs were solved using Mosek v9.2.40. All computations were carried out on Georgia Tech's PACE cluster, where node had a quad-core 2.7 GHz processor and 9 GB of RAM. We consider load variabilities of $\pm 0\%$,

TABLE I

SUMMARY OF CONSTRAINT SCREENING PERFORMANCE FOR ALL THREE FORMULATIONS (PERCENTAGE OF REDUNDANT LINE FLOW LIMITS).

	±0% load variation			**±25% load variation**			**±50% load variation**			**±75% load variation**			**±100% load variation**		
	DC	**QC**	**SDP**	**DC**	**QC**	**SDP**	**DC**	**QC**	**SDP**	**DC**	**QC**	**SDP**	**DC**	**QC**	**SDP**
case3_lmbd	100	66.67	66.67	100	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67
case5_pjm	100	0	16.67	83.33	0	16.67	83.33	0	16.67	83.33	0	16.67	83.33	0	16.67
case14_ieee	100	100	90	100	100	30	100	100	15	100	100	10	100	90	10
case24_ieee_rts	100	86.84	2.63	100	68.42	2.63	100	55.26	2.63	100	39.47	2.63	100	28.95	2.63
case39_epri	97.63	41.30	2.17	86.96	30.43	2.17	69.57	23.91	2.17	67.39	19.57	2.17	56.52	17.39	2.17
case57_ieee	98.75	100	23.75	97.50	96.25	17.50	97.50	96.25	13.75	97.50	92.50	10	97.50	92.50	10
case73_ieee_rts	100	73.33	0.83	100	59.17	1.67	100	44.17	1.67	100	26.67	0.83	100	19.17	0
case118_ieee	96.24	62.37	4.84	93.55	48.92	4.84	87.63	42.47	4.84	83.33	30.11	4.30	79.03	25.81	4.30

$\pm 25\%$, $\pm 50\%$, $\pm 75\%$, and $\pm 100\%$ around the nominal values specified in the test cases.

A. DC vs. QC vs. SDP: Constraint screening performance

For comparing the constraint screening performance between the different OPF formulations considered in this paper, we focus on the removal of line flow limits. This is because many of the other engineering constraints are neglected when formulating the DC approximation. Table I shows the redundant line flow limits for selected test cases. These results include the percentage of redundant line flow limits for different ranges of load variability using the DC approximation, the QC relaxation with bound tightening, and the sparsity-exploiting SDP relaxation.

Among these three formulations, the DC approximation always removed the most line flow constraints. The DC approximation consistently screened out over half of the line flow constraints (in some cases, screening out constraints that were proven to be non-redundant in the full AC case by one of the relaxations), making it by far the least conservative formulation for the purpose of screening out constraints of this type. However, by being a linear approximation and not a convex relaxation of the AC case, redundancy with respect to the DC approximation does not guarantee redundancy for the full nonconvex AC problem.

While both relaxations performed more conservatively than the DC approximation, the QC relaxation removed more constraints than the SDP relaxation for all but one test case (case5_pjm). In several test cases, the QC relaxation outperformed the SDP relaxation considerably. This suggests that the QC relaxation (with bound tightening) is likely tighter than the SDP relaxation for this application.

To further illustrate these results, we show the constraint screening results for case118_ieee graphically in Fig. 3. This figure shows that the DC approximation classifies over 79% of all line flow constraints at all ranges of load variability to be redundant (in blue). The percentage of redundant constraints with the QC relaxation varies between 25% and 62% for different ranges of load variability, while the SDP relaxation classifies less than 5% as redundant. Since the number of constraints characterized as redundant by the nonconvex problem (in yellow) are the same for both the QC and SDP relaxations, the SDP relaxation leaves many more constraints in the category of "inconclusive" (in red).

Not only are the number of redundant constraints different in each formulation, but the particular redundant constraints identified by each also differ. This can be seen in Fig. 4. Even though the QC relaxation formulation screens out many more constraints, this relaxation did not find all redundant constraints identified by the SDP relaxation. Thus, by using both the QC and SDP relaxations, we can increase the number of redundant constraints identified. We also see that not all constraints found to be redundant for the DC approximation are actually redundant in the AC case.

For the remainder of the paper, we will use the QC relaxation with bound tightening to carry out the constraint screening algorithm for the AC OPF problem since (a) using relaxations offers rigorous guarantees of constraint redundancy in the AC case, and (b) the QC relaxation considerably outperformed the SDP relaxation for most test cases. These advantages come at the expense of carrying out the very time consuming bound tightening algorithms, which can become computationally prohibitive for larger cases. The computational tractability of this pre-processing step could be significantly improved by using parallel computing.

B. Analysis of the AC relaxation results

The full results of using the QC relaxation with bound tightening to screen out line flow constraints are shown in Fig. 5. We observe that the percentages of redundant line flow constraints varies greatly depending on test case. While for some test cases, such as case14_ieee and case57_ieee, the algorithm found over 90% of line flow constraints to be redundant for all ranges of load variability, some test cases such as case5_pjm and case39_epri have as few as 0% and 17.39% redundant line flow constraints, respectively.

Moreover, not all types of constraints are removed at the same ratios as line flow constraints. Constraints such as angle difference limits and minimum voltage magnitudes are removed much more consistently throughout all cases. Generator limits, on the other hand are found to be redundant much less frequently. A representative example of this is seen in Table II for case118_ieee. In this case, 100% of lower voltage limits are found to be redundant for all ranges of load variability. However, no more than 18.52% of any given generator constraints are certified as redundant.

C. Computational advantages

Removing redundant constraints reduces the size of optimization problems. In addition to removing the constraints themselves, variables can be removed if all constraints using

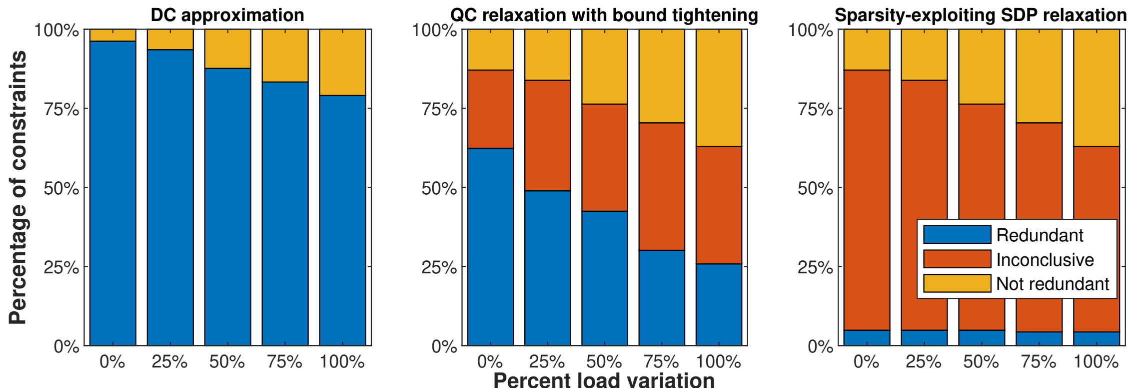

Fig. 3. Redundant line flow constraints for case_118 for the DC approximation, QC relaxation, and SDP relaxation.

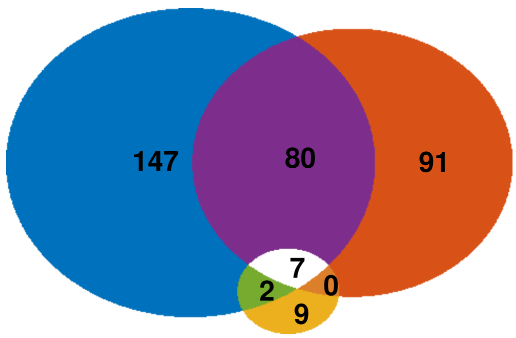

Fig. 4. Venn diagram showing the sets of redundant line flow constraints for case118_ieee with 25% load variability for the DC approximation (blue), QC relaxation (red), and SDP relaxations (yellow), with shared constraints in overlapping colors. The labels show the number of line flow constraints in the corresponding set.

those variables are redundant and the variables are not included in the objective function.

Here we consider the computational advantages of using the constraint screening method from Section III before solving AC OPF problems. After using the QC relaxation to screen out constraints that are guaranteed to be redundant at different ranges of load variability, we run fifty instances of the AC OPF problem with random load profiles within the corresponding variation range. The average solution time is computed and compared to the average time required to solve the original test cases. This comparison is described in Table 6, where we show the time needed to solve the AC OPF before and after removing the redundant constraints. We observe more substantial computational improvements for larger test cases (where the problem size is big) and smaller load variability ranges (where it is easier to remove more variables and constraints to simplify the problem).

V. Conclusion

Power system optimization problems often include variables and constraints that can be safely removed from the formulation. The process of screening out redundant constraints depends largely on the approximations and relaxations applied to the nonconvex power flow equations. This paper compares the constraint screening performance when using the DC approximation, the QC relaxation, and the SDP relaxation. We also evaluate the computational advantages of reducing the size of the optimization problems using this constraint screening method.

The results of this comparison on eight standard test cases suggest that between the QC and SDP relaxations, the QC relaxation provides a tighter relaxation since it permits screening out many more constraints. Furthermore, we demonstrate that the DC approximation incorrectly screens out some constraints (i.e., the DC approximation identifies constraints that may be binding in the AC OPF problem). Lastly, our time comparisons when solving the AC OPF with and without redundant constraints suggest that there are some computational advantages when solving the reduced problem, especially for larger cases and smaller load variabilities.

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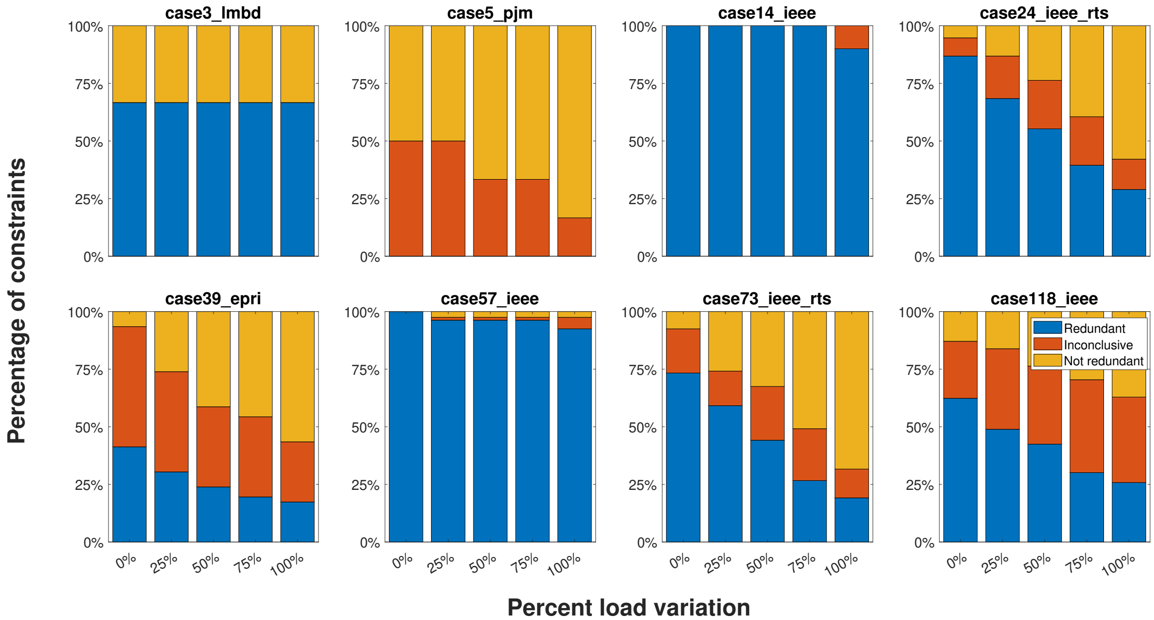

Fig. 5. Percentage of redundant line flow constraints for all test cases at ±0%, ±25%, ±50%, and ±100% load variability.

TABLE II

PERCENT OF REDUNDANT CONSTRAINTS FOR EACH CONSTRAINT TYPE IN CASE_118 USING THE QC RELAXATION.

Load Variation	V^{min}	V^{max}	P^{min}	P^{max}	Q^{min}	Q^{max}	I_{ij}^{max}	$-\theta_{ij}^{max}$	θ_{ij}^{max}
±0%	100	78.81	1.85	0	18.52	12.96	62.37	100	100
±25%	100	73.73	0	0	18.52	11.11	48.92	100	100
±50%	100	66.1	0	0	16.67	7.41	42.47	100	100
±75%	100	48.31	0	0	14.81	7.41	30.11	100	100
±100%	100	22.88	0	0	14.81	7.41	25.81	100	100

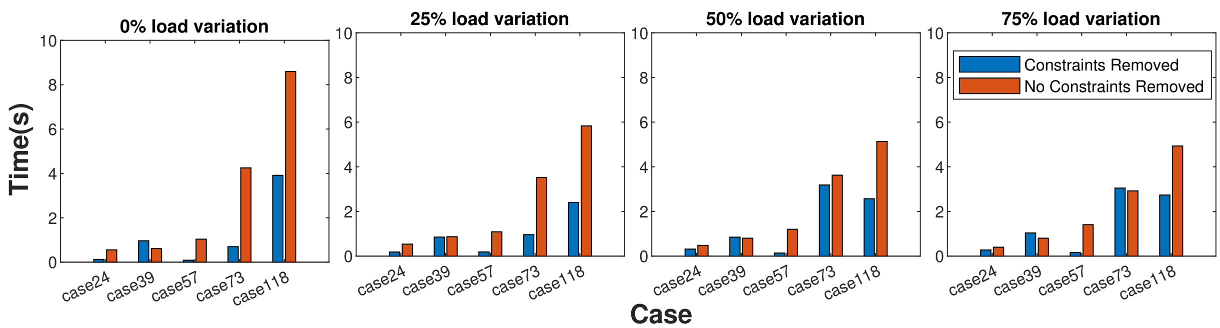

Fig. 6. Computational time with and without redundant constraints removed in representative test cases.

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