Topological and Impedance Element Ranking (TIER) of the Bulk-Power System

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Abstract

The Electricity Modernization Act of the Energy Policy Act of 2005 requires the creation of enforceable reliability standards for users, owners, and operators of the "Bulk-Power System" (BPS). This paper introduces an algorithm based on electrical properties of an electric power network that yields a numeric ranking of its branch elements as a step towards identifying which elements should be considered part of the BPS. This ranking depends solely on network topology and element electrical characteristics, and is wholly independent of generator cost functions and system operating point. Two perspectives for deriving and justifying this ranking algorithm are described: one based on the sensitivity of bus-to-branch Lagrange multipliers and another based on generation shift factors. Summary results obtained from applying the algorithm to a detailed model of the US Eastern Interconnection are reported (with suitable anonymity for any facility specific data, in keeping with Critical Energy Infrastructure Information (CEII) protections).

1. Introduction

On August 8, 2005, the Electricity Modernization Act of 2005 of the Energy Policy Act of 2005 (EPAct 2005), was enacted into law [1]. EPAct 2005 adds a new section 215 to the Federal Power Act (FPA) [2] which requires the Federal Energy Regulatory Commission (FERC) to certify an Electric Reliability Organization (ERO) to develop mandatory and enforceable Reliability Standards, which are subject to Commission review and approval. Once approved, the Reliability Standards are enforced by the ERO, subject to Commission oversight or the Commission can independently enforce Reliability Standards. The Reliability Standards would be applicable to Users, Owners and Operators of the "Bulk-Power System" (BPS). The definition for the Bulk-Power System provided in the statute states

> *The term `bulk-power system' means-- (A) facilities and control systems necessary for operating an interconnected electric energy transmission network (or any portion thereof); and (B) electric energy from generation facilities needed to maintain transmission system reliability. The term does not include facilities used in the local distribution of electric energy.*

This definition includes any elements of the transmission system that are necessary for operating an interconnected electric energy network to achieve Reliable Operation, and specifically excludes local distribution facilities. However, this definition does not directly yield an objective test to classify an element as part of the BPS or not.

At the present, FERC has adopted the following North American Electric Reliability Corporation (NERC) definition for "Bulk Electric System" [3] that employs a specific voltage level for generation, lines, interconnections, and associated equipment as a generally applicable distinguishing metric:

> *Bulk Electric System: As defined by the Regional Reliability Organization, the electrical generation resources, transmission lines, interconnections with neighboring systems, and associated equipment, generally operated at voltages of 100 kV or higher. Radial transmission facilities serving only load with one transmission source are generally not included in this definition.* [4]

The heart of this definition, unless modified by the regions, is a voltage-level threshold: generation, lines, interconnections, and associated equipment operated or connected at voltages above 100 kilovolts (kV) are considered part of the Bulk Electric System, and elements operated at voltages below 100 kV are generally not included, with the exception of interconnection lines. This definition has the clear advantage of being simple to apply; there is no question as to which elements are included. A potential disadvantage of this definition comes from its default disregard for the function of the transmission elements. Some interconnected electric energy transmission networks are built with strong underlying networks at voltages below 100 kV (69 kV being a common voltage), while others will build networks that serve the same function at 115 or 138 kV instead. As a result, much larger portions of the electric system may be included in the Bulk Electric System in some areas, while other areas may have a fairly small fraction of their transmission system included even if both are necessary for the reliable operation of the network.

The largest modification of the BPS definition comes from the Northeast Power Coordinating Council (NPCC). The basic premise of the NPCC definition is that a transmission element should be included in the BPS if the failure of that element causes a significant adverse impact outside of a local area [5]. In practice, this definition leads to the exclusion of many facilities below 230 kV and even some facilities at 230 kV and above.

The Western Electricity Coordinating Council (WECC) offered another definition that outlines a list of circumstances under which an element should be included as part of the BPS [6].

The sampling of currently existing definitions shows the range of definitions and the possible merit in developing a practical, computable numeric ranking that may be used to provide structure in 1) developing a process to distinguish those facilities that should not be considered part of the BPS from those facilities that should be considered part of the BPS, 2) identifying the elements needed to operate each of the electric interconnections, and 3) ranking the importance of those elements. The approach we develop uses a sensitivity analysis to classify elements. We seek to characterize the potential of an individual element to modify or impose network constraints, and in turn, how those constraints impact dispatchable resources in achieving optimal operation. Contingencies are the basis for most constraints and are monitored and controlled in all portions of the grid in order to achieve Reliable Operation. The relative magnitude and additional network locations that are impacted by a contingency on an element provide a practical, objective approach to understanding if that element is needed to enable Reliable Operation of the BPS. In this report we consider as contingencies the limitation of power flow on each element in the network, individually, and rank the elements by their magnitude and spread of impact among dispatchable resources.

Sensitivity analysis is a standard tool in most technical fields. In this paper, we present a sensitivity measure that relates network element constraints to the optimal profile for dispatch. We review properties of constrained optimization problems, and in particular we note that the impact of a constraint is measured by the associated Lagrange multiplier. In the power system context we focus on the network-imposed relation between branch and bus Lagrange multipliers. While the material to follow is most easily understood in the context of a traditional optimization of generator operating cost (or market offer price) in dollars per hour, it is important to stress that the characteristics to be used in this analysis are purely those of the network elements, and are wholly independent of any dollar-valued cost function. However, the approach does assume that the system is being operated in an optimal fashion with respect to *some* objective function, and the commonly used terminology for such an objective is "cost" function. Hence, for ease of understanding, the exposition to follow will use the terminology of minimizing "cost" in a market, so that sensitivities are then characterized in the familiar units of Locational Marginal Prices (LMPs in $/MWhr). We emphasize again that the method does not depend in any way on knowledge of any generator's dollars per hour operating cost or market offer or the existence of a market in a particular portion of the electric system. Indeed, it will be the variation in the pattern of LMPs that are used to compute rankings, rather than specific numeric values of these prices.

We calculate the mathematical sensitivity relating the marginal cost of curtailing flow along an element (branch Lagrange multiplier) to a vector of variation in the marginal cost profile of dispatchable resources in optimal dispatch (variation in bus Lagrange multipliers). While we have not seen this particular problem presented in the literature in this context, there are related works that use similar sensitivity analyses that influenced the choice for this approach. Researchers have developed a market sensitivity approach to identify load pockets and market participants who may have market power potential [7], [8]. The dispatch/price sensitivities allow the identification of market participants who can adjust prices without changing dispatch.

In [9], the relation between branch constraints and LMPs is formally studied. When neglecting losses, LMPs must be uniform (i.e., equal at every generator or dispatchable resource) in the absence of constraints. When deviating from this uniform cost situation, the incremental dispatch profile required to curtail flow along an element imposes a pattern of what are termed "admissible" LMP changes. The exact amount of change realized along this new degree of freedom in the optimal power flow solution depends upon the cost functions for dispatchable resources; however, the *pattern* of LMP changes (i.e., the relative amount of change at each location) does not depend on cost functions.

Using the relation between line elements and admissible LMPs, we perform a sensitivity analysis of the branch Lagrange multiplier to variation of the bus Lagrange multipliers (LMPs). This analysis is then used to rank branch elements by their relative ability to impact variation among LMPs at dispatchable resources.

In the absence of any imposed limit or controls on facilities, optimal dispatch results in generators operating at a point for which the slope of all generators' objective functions are equal (e.g., equal incremental costs). Equivalently, in a market setting, generators operate at equal LMPs. We assert that elements in a local distribution network cannot influence this pattern in the optimal result: generators continue to operate at equal LMPs if a local distribution-only element is curtailed. Conversely, when facilities in the transmission network are curtailed, these elements can impact the pattern of LMPs at an optimal solution.

Using a power system model containing topological information and branch electrical characteristics, we calculate a vector of sensitivities for variation in generator LMPs to the marginal cost of redispatch associated with curtailment of a branch element. The information in the sensitivity vector is condensed to a scalar TIER metric that measures variation in LMPs at optimal dispatch; i.e., this metric characterizes the degree to which the pattern of admissible LMPs departs from the uniform, all equal pattern that must exist at an unconstrained solution. For local radial distribution elements, all their TIER values will equal to zero, identically. For other elements, the TIER value can be used in the ranking and classification of facilities.

2. Mathematical Development

In this section, we derive the TIER metric for ranking branch elements (power lines and transformers) in a power system model. Shunt elements such as shunt capacitors and reactors, SVC's, STATCOM's, etc. are not included.

We consider three desirable properties for the TIER metric: 1) The metric should be computable through a clearly defined algorithm, with practical input data requirements, and should reflect the elements' electrical characteristics and system topology. 2) Any element that only serves radial loads may be considered as having the characteristics of distribution elements, and radial connections to generating plants should have a high importance. 3) The metric should be independent of generator cost functions.

There are two perspectives on the TIER metric, one based on LMP sensitivities and another based on generation shift factors. While the LMP sensitivity formulation has computational advantages and will therefore be used in the systems analyzed in this paper, the two methods give equivalent results.

2.1. LMP Sensitivity Formulation

The LMP sensitivity formulation is based on the DC Optimal Power Flow Model (DCOPF), which is a linear approximation of the more detailed AC Power Flow Model [11]. The key differences between these models are that the DCOPF neglects losses and reactive power.

The DCOPF can be derived from the AC power flow. Consider the active power flow through a branch element

$$P_l = b_l \sin(\theta_m - \theta_n) \approx b_l(\theta_m - \theta_n) \tag{1}$$

where P_l, b_l, θ_m, and θ_n denote the active power flow along the line, the line susceptance (electrical characteristic), and voltage angles at the terminal buses. The more exact nonlinear trigonometric sine function is replaced by its linear "small angle approximation." It is convenient to mathematically represent all the line power flow relations in matrix/vector form

$$P_{flow} = diag(b) A^T \theta \tag{2}$$

where P_{flow} is the vector of power flows along branch elements, θ is a vector of voltage angles at each bus, $diag(b)$ is a diagonal matrix of branch susceptances, and A is a "node-to-branch incidence matrix" [12]. Matrix A describes the connections made by the branch elements, with one row for each bus and one column for each branch element. Each column of A has two nonzero entries: a 1 in the row corresponding to the bus where the branch element begins and a -1 in the row corresponding to the bus where the branch element terminates. The direction is arbitrarily defined.

The DC power flow model relates the power injected at bus locations to bus voltage angles, described mathematically by

$$P_{inj} = AP_{flow} = Adiag(b) A^T \theta \tag{3}$$

This demonstrates the dependence on topology with matrix A and on electrical characteristics with $diag(b)$.

A standard constrained optimization approach is used to optimize the DC power flow problem. The objective function will be denoted as $C(P_g)$, and may be thought of as the production cost of generation. The exact nature of this function will not influence our result. Our problem is minimizing the system cost with the DC power flow constraints and a single additional constraint associated with a branch element (we consider each element independently). For each element our problem is

$$\min_{P_g,\theta} C(P_g) \quad \text{subject to}$$

$$P_{inj} = A\,diag(b)\,A^T\theta \quad \text{and} \tag{4}$$

$$P_{line} = b_{line}A_{line}^T\theta$$

where P_{line} is a scalar limit imposed on the branch element power flow, b_{line} is the branch susceptance, and A_{line} is the column of A corresponding to the constrained element.

This problem can be solved using the classic method of Lagrange multipliers. The Lagrange function is written for this problem as

$$\begin{aligned} L(P_g,\theta) = C(P_g) + \lambda^T\left(Adiag(b)A^T\theta - P_{inj}\right) \\ + \mu_{line}\left(b_{line}A_{line}^T\theta - P_{line}\right) \end{aligned} \tag{5}$$

In (5), both λ and μ_{line} are Lagrange multipliers associated with the constraints. In economic terms, λ represents a vector of "shadow prices" of each bus constraint. In power systems terms, this vector λ contains the locational marginal price at each bus in the system. Similarly, μ can be thought of as the shadow price of the line's power (i.e., the incremental cost of curtailing the line's flow by one megawatt). The values of Lagrange multipliers provide a metric with which to compare the impact of line constraints.

Setting the derivatives of the Lagrange function equal to zero will yield the Karush-Kuhn-Tucker necessary conditions for optimality that must be satisfied at any solution of the constrained optimization problem. The relationship between λ and μ_{line} comes from a subset of these conditions that are associated purely with the network's behavior. In particular, we employ the condition arising from the derivative with respect to θ being set to zero:

$$\frac{\partial L(P_g,\theta)}{\partial\theta} = Adiag(b)A^T\lambda + A_{line}b_{line}\mu_{line} = 0 \tag{6}$$

From this equation, it is possible to obtain a relationship between λ and μ_{line} which does not depend on the generator cost functions $C(P_g)$. Solving (6) for the variation in λ (suppressing a common uniform component) in terms of μ_{line} results in the profile of LMP sensitivities which we use to rank model elements.

To understand the possible solutions to (6), it is instructive to consider the equation with no constraints:

$$\frac{\partial L(P_g,\theta)}{\partial\theta} = Adiag(b)A^T\lambda = 0 \tag{7}$$

Because the matrix $A\,diag(b)\,A^T$ is singular, there is a meaningful nontrivial solution to this equation

$$\lambda = \text{constant} \tag{8}$$

In the context of power system economic dispatch, this constant would be the value of equal marginal costs, or in a market setting, uniform LMPs. Additional information about costs (market offers) would be necessary to determine the value for the constant λ; however, the sensitivities we derive do not depend on cost functions.

The constraint in (6) adds an additional degree of freedom to the possible solutions

$$\lambda = \text{constant} + \lambda_T \tag{9}$$

where λ_T is a zero-sum vector that is orthogonal to the constant vector. While we cannot compute the values for the constant or for λ_T, we can determine the pattern imposed on the solution: the constant is a uniform vector of equal values and λ_T is a zero-sum, non-uniform vector that takes on a computable pattern of values scaled by the line Lagrange multiplier μ_{line}. We seek to calculate the pattern of values for the non-uniform vector λ_T. This pattern is the sensitivity of the variation in bus Lagrange multipliers (λ_T) to line Lagrange multipliers (μ_{line}).

To solve for λ_T in terms of μ_{line}, we augment the constraint equation with an equation that brings out only the solution that is orthogonal to any constant vector

$$\begin{bmatrix} Adiag(b)A^T & A_{line}b_{line} \\ 1^T & 0 \end{bmatrix}\begin{bmatrix} \lambda^T \\ \mu_{line} \end{bmatrix} = 0 \tag{10}$$

where 1^T is a row vector of all ones.

The solution to (10) can be easily obtained using tools for solving linear algebraic equations, and can be easily applied to very large-scale power networks using sparse matrix techniques.

In the case where there are no constraints, the flow on any element can change freely and all LMPs are equal. When the flow on an element is curtailed, its associated Lagrange multiplier will not be zero and variation in bus Lagrange multipliers, the LMPs, will arise across the network. Although the full solution for LMPs cannot be determined without generator cost data, the LMP variation profile can be determined from (10). The effect on generation dispatch can be compared by examining the effect of LMPs at generator buses. If a flow curtailment on a certain transmission element affects all generator LMPs equally, then the curtailment has no effect on generation dispatch profile. Conversely, a curtailment that affects generators unequally will have an impact on generation dispatch. In this case, the system dispatch loses a degree of freedom and the possible

LMPs gain a degree of freedom. In mathematical optimization terminology, the primal variables (P_g and θ) lose one degree of freedom, while the dual variables (LMPs) gain a degree of freedom.

The LMP variation sensitivity λ_T from (10) has entries associated with each bus in the system. Since practical power systems can have thousands of buses, the full λ_T vector has an impractically large amount of data to use for ranking. Our goal is to characterize in a single numeric quantity how far the system has deviated from the unconstrained case of equal LMPs. Therefore, the TIER method calculates the standard deviation of values in the vector of LMPs associated only with dispatchable resources (typically generators, though controllable, dispatchable loads can also be incorporated). This provides a practical, easily computable scalar metric to rank elements. The standard deviation is a 2-norm of the deviations of the Lagrange multipliers associated with dispatchable resources away from their mean. Other common norms we considered include the 1-norm (the sum of the absolute values) and the infinity-norm (the largest absolute value). However, the 1-norm fails to consider large local impacts, such as a line which is the only connection from the majority of the system to a number of generating units. Conversely, the infinity-norm under-emphasizes elements which cause modest impact over most of the transmission system. The two-norm balances these two situations well. Denoting λ_{TIER} to be the subset of λ_T associated with dispatchable resources, the TIER value for a branch element is defined to be

$$\mathrm{TIER}_{\mathrm{line}} = std\left(\frac{\lambda_{TIER}}{\mu_{line}}\right) \tag{11}$$

where *std* denotes standard deviation.

2.2. Generation Shift Factor Formulation

For the reader familiar with electricity markets, but less familiar with the role of Lagrange multipliers in optimization theory, the previous development for the TIER may appear to only apply in a market setting, despite our comments to the contrary. This is not the case and we present next another formulation of the TIER method using generation shift factors. Shift factors are commonly used to give a linear approximation of changes in generator dispatch to changes in transmission element flows. Shift factors allow for an alternate DC power flow model that eliminates the set of phase angle decision variables (θ). A shift factor matrix S with one row for each transmission element and one column for each generator is defined using the formula

$$S_{\mathrm{lg}} = \frac{\Delta P_l}{\Delta P_g} \tag{12}$$

l represents the index of the branch element in question and g represents the index of a generator. ΔP_l represents the change in power flow on line l caused by a change of ΔP_g in the generation of generator g. Using S, we can rewrite (4)

$$\begin{aligned} &\min_{P_g} C(P_g) \quad \text{subject to} \\ &\sum P_g = \sum P_d \\ &\left| S\, P_g \right| \le P_{\mathrm{limit}} \end{aligned} \tag{13}$$

The first constraint of (13) is a system-wide balance of power. The second constraint reflects the fact that the real power flow on each branch element must remain within its limit.

As with the original formulation, we examine the hypothetical situation where exactly one element (referred to as element k) is constrained.

$$\begin{aligned} &\min_{P_g} C(P_g) \quad \text{subject to} \\ &\sum P_g = \sum P_d \\ &S_k P_g = P_{\mathrm{limit,\,k}} \end{aligned} \tag{14}$$

S_k represents the k^{th} row of the matrix S. This vector is equal to the vector of LMP variation sensitivities calculated through the original derivation (λ_T) except for a reference-bus dependent constant offset. Since we are eventually taking the standard deviation of the entries in this vector, the constant offset actually will have no effect on the final results. Using this formulation, the definition of the TIER value for the k^{th} line is given by

$$\mathrm{TIER}_{\mathrm{k}} = std\left(S_k\right) \tag{15}$$

where *std* denotes standard deviation.

This approach offers additional intuition into the TIER values. Any line which has widely varying shift factors will have a fairly high TIER value, while any line with zero (or almost zero) shift factors for every generator will have a zero (or very low) TIER value.

Computational issues make the shift factor formulation more difficult to implement than the LMP sensitivity formulation. To calculate shift factors, the output of each generator must be varied, one at a time, to determine the effect on transmission element flows. However, to calculate the TIER values, we need the effect of each generator on only one line. Therefore, we would need to calculate the entire S matrix before calculating TIER values. The S matrix is not sparse and the memory requirements can become prohibitively large for practical power systems. Conversely, the line-by-line approach used in the LMP sensitivity formulation enables us to calculate one line's sensitivities, calculate the TIER value, and then immediately discard the sensitivities to conserve memory.

As mentioned earlier, the shift factor vectors are only a constant offset away from being exactly equal to the LMP variation sensitivity vectors. This constant, which is different for each transmission element, is dependent on the choice of the system reference bus. Since the standard deviation calculation is invariant to a constant offset, the choice of reference bus has no effect on the final TIER value results.

3. Example

To illustrate the TIER method, we apply the LMP variation sensitivity formulation to the small system shown in figure 1.

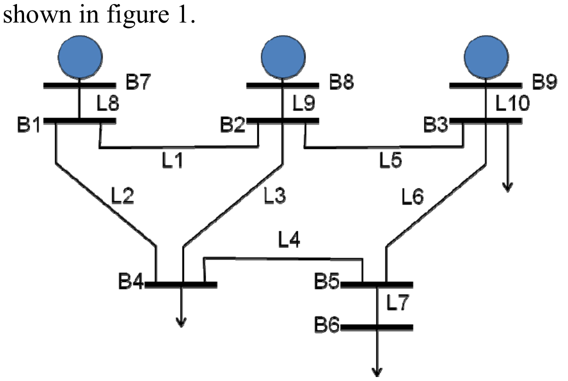

Figure 1. A small example system

The node-to-branch incidence matrix A, which contains the system topological information, for this system is

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

In this example, we set all susceptance values to 10, so $diag(b)$ is a diagonal matrix with values of 10 on all diagonal elements. With uniform electrical characteristics, all differences in results may be attributed to system topology.

The TIER value calculation for line L1 will be performed in detail. Solving (10), where A_{line} is the first column of A, gives the variation of λ in terms of μ_{L1}.

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \end{bmatrix} = \begin{bmatrix} -0.4040 \\ 0.2323 \\ 0.1414 \\ -0.0404 \\ 0.0505 \\ 0.0505 \\ -0.4040 \\ 0.2323 \\ 0.1414 \end{bmatrix} \mu_{L1}$$

The TIER value for L1 is calculated as the standard deviation of three entries of this vector that correspond to generator buses. These are the entries for λ_7, λ_8, and λ_9. The TIER value is

$$\sigma_1 = std(-0.4040, 0.2323, 0.1414) = 0.344$$

Repeating the calculations for the remaining lines yields the following results

Table 1. TIER values for small example system

Line number	TIER value	Rank
1	0.344	5
2	0.241	6
3	0.139	9
4	0.189	7 (tied)
5	0.396	4
6	0.189	7 (tied)
7	0.000	10
8	0.577	1 (tied)
9	0.577	1 (tied)
10	0.577	1 (tied)

There are three important observations from these results. First, the radial transmission element L7 that connects loads to the rest of the system has a TIER value of zero. This is entirely a result of topology, and is consistent with the Bulk-Power System definitions reviewed in Section 1 that require radially connected elements that only serve loads to be classified as distribution elements.

Second, the TIER values for radial elements that connect generators to the rest of the transmission system (lines 8, 9, and 10) are equal. This is also a result of topology. Although these are the most important elements in this small example system, radial generator connections will tend to become less important than the high-voltage backbone in larger systems.

Third, transmission elements that primarily connect generators (lines 1 and 5) are ranked fairly high. Elements that are closer to only loads (line 4) tend to be ranked lower.

4. Results

The TIER method was applied to a detailed model of the Eastern Interconnection of the US electric grid. While TIER values are automatically calculated for all elements in the model, we only discuss those elements within the USA. These results are from an actual planning model used in industry.

This model consists of nearly 50,000 buses and 60,000 branch elements in the United States. It is interesting to note that a significant percentage of the facilities in this model would not be included in the NERC definition for Bulk Electric System: slightly more than one half of the bus voltages and more than one third of the branch elements are below 100 kV.

The computation of TIER values for all elements in the model took approximately one hour on a laptop of modest computing power using MATLAB software. The vast majority of computational effort is consumed calculating the null space of a large matrix. The rankings appeared intuitively reasonable. Qualitative trends expected of the rankings were observed; for example, high-voltage elements were generally ranked more highly than low-voltage elements. However, for instances in which the rank assigned a network element did not follow the expected voltage trend, further scrutiny revealed sound reasons grounded in the network topology that justified the high or low TIER value.

One of the qualitative features most expected, and confirmed by the analysis results, was that higher-voltage lines should typically have higher importance values. Clearly, the extra-high voltage (EHV) backbone of any system is generally considered one of the most important parts of the system. The low per-unit impedance and high thermal ratings of EHV elements means that power flowing over long distances will tend to predominately use these elements. Table 2 shows the trend of increasing TIER values with higher voltage. For this table, any transformers are listed under the highest voltage level to which they are connected. For example, a 345-115 kV transformer is included in the 345 kV category. Generator step-up transformers and all connections to radial loads are excluded.

Table 2. TIER values vs. voltage

Voltage	Lowest	Average	Highest
765 kV	0.0032	0.0354	0.1022
500 kV	0.0015	0.0264	0.1480
345 kV	7.3×10^{-9}	0.0159	0.1343
220/230 kV	9.6×10^{-10}	0.0085	0.0761
100 – 199 kV	1.5×10^{-10}	0.0049	0.0602
51 – 99 kV	1.0×10^{-11}	0.0027	0.0279
≤ 50 kV	4.8×10^{-10}	0.0042	0.0279

While the average TIER value tends to increase as the operating voltage increases, confirming the "typically" expected trend, the range of importance values among elements within a given voltage level is relatively large. This leads to substantial overlap between TIER values assigned to elements across different voltage levels; for example, a significant fraction of the elements operated at 345 kV are ranked above a number of the elements operated at 500 and 765 kV. This shows that the topology of a system can have a significant effect on the importance of a network element, regardless of the voltage level at which it is built and operated.

A property of the results that is potentially relevant to classification can be seen in the distribution of the TIER values. Figure 2 shows the TIER values on a base-ten logarithmic vertical scale versus the element rank on the horizontal scale. The logarithmic vertical scale means that elements with zero importance value (associated with lines serving radial loads) are off the scale towards the negative. Generator step-up transformers are not included in this TIER ranking.

Additional structural features of the method can be observed from this graph. First, the most important elements, which are located on the far left side of the graph, span a fairly wide range of TIER values (approximately an order of magnitude on the logarithmic scale). Second, there is a steep drop-off in TIER value for those elements ranked in the range of roughly 42,100 to 43,700. This indicates that there is a fairly small percentage of elements (fewer than 3%) in the system having TIER values below 10^{-4} but greater than the "hard" zero value that is assigned to radial load serving elements. And there are fewer than 600 elements (approximately 1%) with a TIER value between 10^{-5} and zero.

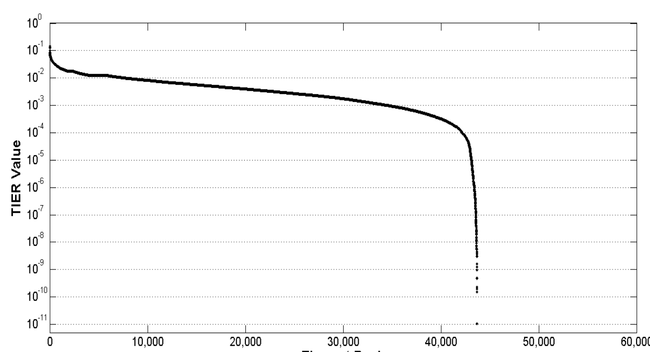

Figure 2. TIER value vs. rank

As a different view of the data shown above, figure 3 displays the density function of TIER values (i.e., the vertical axis displays the relative frequency of occurrence, with respect to the TIER values displayed on the horizontal axis). To enhance readability, the data set is filtered before constructing the density plot. In particular, data associated with radial load elements, that would have contributed a large "spike" at a zero TIER value, is uninformative and is not included in the plot. Likewise, data for elements that form radial connections to generators, which all yield identically equal TIER values, is also eliminated. The resulting plot then clearly displays the same trend as figure 2: a few very high TIER values, a relatively large number of TIER values between about 10^{-1} and 10^{-4}, and a very small number of elements with TIER values below that range.

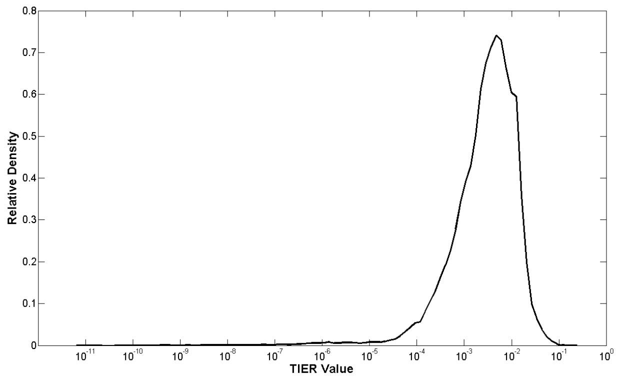

Figure 3. Distribution of TIER values

Finally, as a way of interpreting a combination of results shown in the table of voltage levels and in the previous graphs, one can plot the relative distribution of TIER values at each voltage level separately. This gives a graphical representation of the general trends observed in the results: as an average across the group, elements with higher operating voltages have higher TIER values, but the variation within a voltage level is large enough to provide significant overlaps between voltages. As with the previous graphs, the TIER values here are plotted on a base-ten logarithmic scale. The plots were created with MATLAB's *ksdensity* function, which creates a smooth density function such that the area under the function integrates to exactly 1. The curves were then scaled by multiplying each curve by the number of elements in that voltage class. The TIER values for lines and transformers are presented in figure 4. For the purposes of distinguishing by voltage, transformers are categorized by their highest voltage level. For example, a 500-138 kV transformer is categorized with the group 500/765 kV. For these plots, radial connections to loads and generators were eliminated.

The TIER values observed in figure 4 show a general correlation to voltage level, with the higher voltage elements tending to have higher TIER values. However, the overlap in distributions confirms that any partitioning of elements by voltage level will differ considerably from partitioning of elements by TIER value.

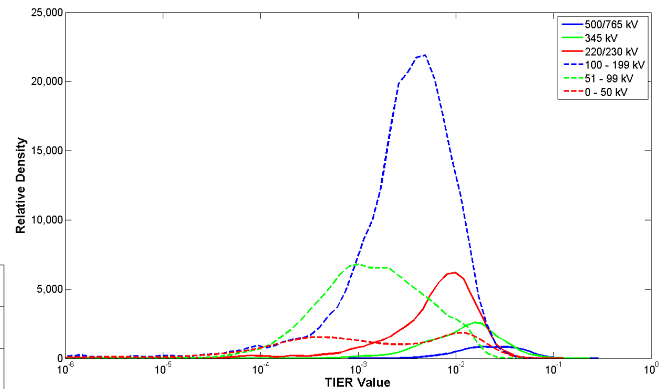

Figure 4. Distribution of TIER values by voltage

5. Discussion

There are several important issues relevant to TIER. First, for an element to be ranked, it must explicitly appear in the model. Equivalencing methods are sometimes used to approximate multiple physical components by a smaller number of fictitious elements in the model. Such equivalencing undermines the objectives of this work. The TIER method relies on a suitably complete and detailed network model as input. Even if a detailed representation of the network section of interest is present, the approximation and errors associated with equivalancing outside the area of interest can propagate to perturb rankings computed for "non-equivalenced" elements located near the edge of the network section of interest. These elements tend to be ranked lower than they would otherwise have been because equivalencing eliminates the possibility of loop flows. The most straightforward way to avoid this issue is to use a detailed model of the entire interconnect, as was done with the Eastern Interconnect model in this paper.

A second issue regards the comparison of TIER values between different systems. TIER values are intended as a relative measure in comparison with other elements in the same system. Inherent to the TIER algorithm is the fact that the absolute importance value of an individual element will tend to be smaller if that same element is placed within a larger system. This simply reflects the physical reality that one single element will typically have smaller absolute impact on the overall system if that system is composed of a very large number of power carrying paths. This property will have no effect on comparing the importance of network elements in the same system. However, with the scaling methods used in this analysis, importance value comparisons from one system to another are not valid.

A third issue involves the robustness of the mathematical model. In computing TIER values, we have chosen the simplification of the DC power flow approximation to represent the network electrical behavior. The "small angle" approximation used in the DC power flow is a widely used and valid approximation, but neglects potential impact of changes in system operating point. We were initially concerned that a potentially more accurate linearization about the exact power flow solution would result in significantly changed TIER values relative to the simpler small angle approximation. However, as shown in [10], empirical evidence using a PJM system model indicates little difference between the two calculations. This reinforces the appropriateness and validity of the DC power flow approximation and indicates that the TIER values can be calculated quite accurately without the need for the data of a full power flow solution at a particular operating point.

A fourth issue that must be addressed for many practical system models regards actively controlled series elements. Actively controlled series elements that effectively respond in real time were treated as active power dispatchable resources. In our analysis, high voltage DC lines and variable frequency transformers were treated in this way. Phase shifting transformers, which can be adjusted but are not typically continuously controlled, were modeled as series impedances. If warranted, some phase shifting transformers could be treated as dispatchable resources depending on the installation and its controls.

A fifth issue in using TIER involves changes in in-service status. The TIER value for each element may change as the system configuration changes with lines and generators going into and out of service. Given the low computational cost of the TIER method, it is reasonable to suggest that the TIER rankings obtained from a number of representative system configurations could be used in concert to judge which elements belong to the Bulk-Power System. For the binary task of assigning an element as part of the BPS, or not, the facility should be so assigned if it is identified to be part of the BPS under any credible operating status.

Finally, TIER does not attempt to create a comprehensive list of all facilities that should be considered part of the BPS. Rather, the facilities identified as important by TIER construe a minimum set of important facilities. Further investigation will be necessary to identify other important facilities. For instance, shunt elements such as Static VAR Compensators are not included in TIER. It is likely that the classification of many of such facilities as part of the BPS will be obvious and consistent with the classification of the neighboring connected branch elements. Other criteria to identify additional facilities to include in the BPS may also be used. For example, the phenomenon of Fault-Induced Delayed Voltage Recovery has been the subject of a recent NERC whitepaper [13]. This is a load-driven phenomenon that can impact the transmission grid following a brief (and normally cleared) fault. Detailed analyses of this phenomenon might argue for the inclusion in the BPS of additional facilities that predominately support load and that may not be highly ranked by TIER.

6. Conclusion

We have presented a practical, objective, computable numeric method for ranking branch elements in a power system model relative to one set of actions necessary to assure Reliable Operation of the interconnected electric energy transmission network. The chosen action is based on generation redispatch and is measured in terms of the sensitivity of bus-to-branch Lagrange multipliers, or locational marginal prices, to a marginal cost of redispatch in controlling flow along an element. A standard deviation to measure variation from uniform marginal cost dispatch (uniform LMP for markets) for dispatchable resources is used for a scalar metric which is defined as Topological and Impedance Element Ranking (TIER). TIER values for radial loads will be zero. We emphasize the following:

- The model and calculation employ only topological information about the network and electrical characteristics of the elements.
- The analysis does not require any knowledge of specific cost functions.
- The analysis may be performed without data on a specific operating point of the system (i.e., full power flow solution is not required).
- The calculations are straightforward, of modest computational cost, and can be performed using ordinary consumer level computers and MATLAB software.

- Generation shift factors provide an alternate perspective to interpret the TIER values.

We have applied this method to several large system models, including the US Eastern Interconnect model detailed in this paper. A typical distribution of importance rankings is characterized by a sharp transition region corresponding to relatively few elements separating the zero-valued connections to radial loads and a region with elements that have intermediate TIER value. We also note that while the TIER value for typical elements tends to decline on average with voltage level, there is significant overlap in TIER values between voltage levels. Therefore, partitioning of elements by voltage level will differ considerably from partitioning of elements by TIER value. System topology plays an important role in determining TIER values, and topology inherently is *not* reflected in any voltage-based classification.

Further research is warranted to consider normalizing the TIER metric to allow for absolute comparisons between different networks. Future research could also focus on application of the TIER metric to study critical system facilities. In the present work, we focus on distinguishing distribution elements at the low end of the TIER scale. It would be valuable to compare how the metric ranks critical elements at the high end relative to other impact-based analyses.

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