

# Improving Distribution System Resilience by Undergrounding Lines and Deploying Mobile Generators

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## Abstract

To improve the resilience of electric distribution systems, this paper proposes a stochastic multi-period mixed-integer linear programming model that determines where to underground new distribution lines and how to coordinate mobile generators in order to serve critical loads during extreme events. The proposed model represents the service restoration process using the linearized DistFlow approximation of the AC power flow equations as well as binary variables for the undergrounding decisions of the lines, the configurations of switches, and the locations of mobile generators during each time period. The model also enforces a radial configuration of the distribution network and considers the transportation times needed to deploy the mobile generators. It is shown that long-term line undergrounding decisions which are cognizant of short-term mobile generator deployments yield superior results relative to undergrounding decisions made without considering mobile generators. Using an extended version of the IEEE 123-bus test system, numerical simulations show that combining the ability to underground distribution lines with the deployment of mobile generators can significantly improve the resilience of the power supply to critical loads.

*Keywords:* Power distribution resilience, mobile generators, undergrounding, service restoration, natural disasters.

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## 1. Introduction

Natural disasters have caused large-scale power outages in recent years, with the total cost of 308 major natural disasters since 1980 exceeding \$2 trillion in the United States alone [1]. Accordingly, the resilience of distribution systems has become a topic of substantial interest for researchers and distribution utilities. The National Infrastructure Advisory Council (NIAC) states that resilience is “the ability to reduce the magnitude and/or duration of disruptive events. The effectiveness of a resilient infrastructure or enterprise depends upon its ability to anticipate, absorb, adapt to, and/or rapidly recover from a potentially disruptive event” [2]. In 2011, the UK Energy Research Center provided a similar definition for the resilience as “the capacity of an energy system to tolerate disturbance and to continue to deliver affordable energy services to consumers. A resilient energy system can speedily recover from shocks and can provide alternative means of satisfying energy service needs in the event of changed external circumstances” [3]. Survivability and swift restoration capabilities are the key features of resilient distribution systems.

Resilience can be improved by reducing the initial impact of a disaster and quickly restoring the supply of power after a disaster occurs. In the literature, two different approaches have been proposed to enhance distribution system resilience, namely, infrastructure hardening and smart operational strategies [4]. Hardening strategies reduce the impacts of disasters. For example, the approach in [5] identifies optimal locations to harden lines and place switches in order to reduce the effects of high-impact low-probability (HILP) events and enhance the restoration performance of the system. The study in [6] proposes a robust line hardening strategy against the worst  $N - k$  contingencies. In this strategy, the objective is to minimize the cost of the operation of multiple islanded provisional microgrids and cost of line hardening. Also, the authors in [7] deployed line hardening and distributed generator (DG) placement measures to minimize the cost of load shedding and aggregated vehicle travel time. A battery sit-

30 ing and sizing strategy for an islanded hybrid AC/DC microgrid is developed  
in [8], in which the objective is to improve voltage and frequency regulation  
performance while considering the post-contingency corrective rescheduling. As  
another example, the authors of [9] enhance distribution system resilience by  
combining line hardening, installing distributed generators, and adding remote-  
35 controlled switches.

On the other hand, smart operational strategies mainly focus on the rapid  
recovery of distribution systems by reconfiguring the network and using avail-  
able resources to provide energy to disconnected loads after an extreme event.  
As an example of smart operational strategies, our previous work in [10] pro-  
40 poses a three-stage service restoration model for improving distribution system  
resilience using pre-event reconfiguration and post-event restoration considering  
both remote-controlled switches and manual switches. Additionally, the authors  
of [11] propose a proactive management scheme for microgrids to cope with se-  
vere windstorms. This scheme minimizes the number of vulnerable branches in  
45 service while the total loads are supplied. Also, the study in [12] presents a two-  
stage stochastic programming method for the optimal scheduling of microgrids  
in the face of HILP events considering uncertainties associated with wind gen-  
eration, electric vehicles, and real-time market prices. For further examples of  
both infrastructure hardening and smart operational strategies, see [13, 14, 15].

50 Recent research has also studied distribution system restoration problems  
that consider the deployment of mobile generators [16, 11, 17, 12, 18]. In [16], we  
proposed a two-stage strategy involving 1) a preparation stage that reconfigures  
the distribution system and pre-positions repair crews and mobile generators  
and 2) a post-HILP stage that solves a stochastic mixed-integer linear program-  
55 ming (MILP) model to restore the system using distributed generators, mobile  
generators, and reconfiguration of switches. Building on this existing work, this  
paper proposes a resilience enhancement strategy that aims to supply power  
to critical loads by selectively adding new underground distribution lines prior  
to a disaster while considering the capabilities of mobile generators to restore  
60 power during the initial hours of a disaster. The mathematical formulation in

this paper considers two time periods: a long-term planning phase that chooses locations for new underground lines and a short-term restoration phase where mobile generators are dispatched and switching configurations are selected to minimize power outages. The new underground lines selected in the planning  
65 phase are chosen in a manner that is cognizant of the actions in the short-term restoration phase. Thus, the mathematical formulation is useful for both long-term planning time periods (where the binary choices for undergrounded lines are variables) and short-term restoration time periods (where the binary choices for line undergrounding decisions are fixed) in order to determine the dispatches  
70 of mobile generators and switch configurations. We will show that long-term line undergrounding decisions which are cognizant of short-term mobile generator deployments yield superior results relative to undergrounding decisions made without considering mobile generators. In this study, the infrastructure recovery phase (i.e., repairing the damaged components) is not included in the  
75 model because the complexity of this process significantly increases the computational burden of the model. We next motivate the use of underground lines and mobile generators for improving distribution system resilience.

***Connecting Critical Loads by Underground Lines:*** Critical loads, such as hospitals, water pumping facilities, and emergency shelters, are pri-  
80 oritized for energization after extreme events. Improving the connectivity of critical loads can increase the resilience of their power supply. There are various advantages and disadvantages to adding new connections via overhead versus underground distribution lines. For example, compared to overhead distribu-  
85 tion lines, underground lines provide reduced likelihood of damage during natural disasters and improved aesthetics [19]. However, undergrounding all of the distribution lines can be prohibitively expensive and also has drawbacks such as utility employee work hazards during faults and manhole inspections as well as potential susceptibility to flooding and storm surges. To take advantage of the reduced susceptibility of underground distribution lines to extreme events while  
90 avoiding their extra costs and potential downsides, we focus on adding new underground lines that support power supply to critical loads. Our proposed

strategy determines the location of new underground lines in order to improve resilience over a range of disaster scenarios while considering the capabilities of mobile generators during the restoration process.

95     **Mobile Generators:** Mobile generators are increasingly being used by utility companies [20, 21, 22]. Careful deployments of mobile generators can significantly enhance the resilience of distribution systems [18]. In this paper, mobile generators are employed to improve the restoration process in the aftermath of HILP events. Since they can be connected to various locations, mobile  
100 generators provide significant flexibility for responding to an extreme event.

To summarize, the need to improve distribution system resilience motivates stronger connections between critical loads as well as more reliable and flexible power sources. When utilized appropriately, underground distribution lines and mobile generators can address these needs. To this end, the main contributions  
105 of this paper are:

- Formulating a stochastic multiperiod optimization model for distribution system restoration that jointly considers the ability to underground lines, reconfigure the network, and reposition mobile generators in order to serve critical loads. This model has applications in both choosing lines to under-  
110 ground for long-term planning purposes and optimizing short-term operational decisions for network reconfiguration, mobile generator deployment, and use of distributed energy resources during extreme events.
- Numerically demonstrating the value of the proposed optimization model relative to two alternative approaches, with the results showing significant  
115 benefits for choosing line long-term undergrounding locations in a manner that is cognizant of short-term mobile generator operations across many extreme event scenarios.

The remainder of this paper is organized as follows: Section 2 describes mathematical formulations of the stochastic MILP model. To demonstrate this  
120 model, Section 3 presents a case study and simulation results. Finally, Section 4 concludes the paper and discusses future work.

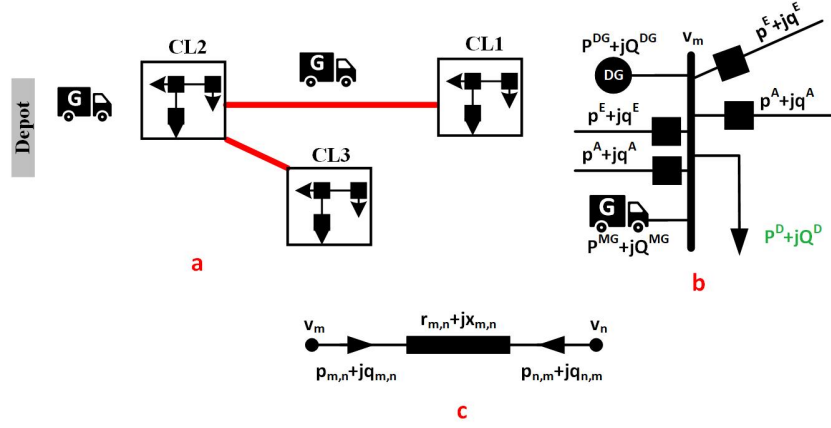


Figure 1: a) Mobile generator and underground line models, b) bus model, and c) line model.

## 2. Mathematical Formulation

This section formulates the proposed stochastic MILP model for the system restoration problem considered in this paper. Fig. 1 shows a sketch of the proposed strategy along with the line and bus models. Define the sets  $\Pi$ ,  $\mathcal{N}$  and  $\mathcal{T}$ , respectively, corresponding to the damage scenarios, the system's buses, and the time periods for the restoration problem. Let subscripts  $\pi$ ,  $t$ , and  $m$  denote a particular scenario, time, and bus, respectively. Each time period has a duration of  $\Delta t$  (e.g., 5 minutes) and we consider a horizon lasting a few hours after the disaster. The load at each bus  $m$  has an assigned weight  $\omega_m$  indicating its importance. The variable  $P_{\pi,t,m}^{LC}$  denotes the curtailment of active power load at bus  $m$  and time  $t$  under scenario  $\pi$ . The probability of each scenario is denoted by  $pr_{\pi}$ . The objective (1) minimizes the total unserved energy during the restoration process, weighted by the load's importance:

$$\min \sum_{\pi \in \Pi} \sum_{m \in \mathcal{N}} \sum_{t \in \mathcal{T}} \omega_m \cdot pr_{\pi} \cdot P_{\pi,m,t}^{LC} \cdot \Delta t \quad (1)$$

### 2.1. Power Flow

Let  $\mathcal{L}^E$ ,  $\mathcal{L}^A$ , and  $\mathcal{L} = \mathcal{L}^E \cup \mathcal{L}^A$  denote the set of existing lines, the set of added underground lines, and the set of all lines, respectively. In this study, we

consider a balanced single-phase equivalent model of the distribution systems, but the approach could be generalized to unbalanced three-phase network models as well. Let parameters  $P_{t,m}^D$  and  $Q_{t,m}^D$  denote the active and reactive power demand at bus  $m$  at time  $t$ , respectively. We also define the following variables for each time period  $t$  and scenario  $\pi$ :

- $p_{\pi,t,m,n}^E$  and  $q_{\pi,t,m,n}^E$ : active and reactive power flows on the existing overhead line connecting buses  $m$  and  $n$ ,
- $p_{\pi,t,m,n}^A$  and  $q_{\pi,t,m,n}^A$ : the active and reactive power flows on the added underground line connecting buses  $m$  and  $n$ ,
- $P_{\pi,t,m}^{DG}$  and  $Q_{\pi,t,m}^{DG}$ : active and reactive power of the distributed generator at bus  $m$ ,
- $P_{\pi,t,m}^{MG}$  and  $Q_{\pi,t,m}^{MG}$ : active and reactive power of the mobile generator at bus  $m$ .

The power balance constraints at each bus  $m$  are:

$$P_{t,m}^D + \sum_{\substack{(i,n) \in \mathcal{L}^E \\ \text{s.t. } i=m}} p_{\pi,t,m,n}^E + \sum_{n \in \mathcal{N}} p_{\pi,t,m,n}^A = P_{\pi,t,m}^{LC} + P_{\pi,t,m}^{DG} + P_{\pi,t,m}^{MG} + \sum_{\substack{(m,i) \in \mathcal{L}^E \\ \text{s.t. } i=n}} p_{\pi,t,n,m}^E \quad (2a)$$

$$Q_{t,m}^D + \sum_{\substack{(i,n) \in \mathcal{L}^E \\ \text{s.t. } i=m}} q_{\pi,t,m,n}^E + \sum_{n \in \mathcal{N}} q_{\pi,t,m,n}^A = Q_{\pi,t,m}^{LC} + Q_{\pi,t,m}^{DG} + Q_{\pi,t,m}^{MG} + \sum_{\substack{(m,i) \in \mathcal{L}^E \\ \text{s.t. } i=n}} q_{\pi,t,n,m}^E \quad (2b)$$

Equation (3a) implies that load curtailment at each bus cannot exceed the total demand of the bus. We also consider a constant power factor load curtailment model as indicated by (3b).

$$0 \leq P_{\pi,t,m}^{LC} \leq P_{t,m}^D \quad (3a)$$

$$Q_{\pi,t,m}^{LC} \cdot P_{t,m}^D = P_{\pi,t,m}^{LC} \cdot Q_{t,m}^D \quad (3b)$$

Let  $v_{\pi,t,n}$  denote the voltage magnitude at bus  $n$  during time  $t$  in scenario  $\pi$ .  $M$  is a big-M constant. Let  $r_{m,n}, x_{m,n}$  denote the resistance and reactance of line  $(m, n)$ .  $\varsigma_{\pi,t,m,n}$  is a binary variable indicating whether the switch on the line  $(m, n)$  at time  $t$  in scenario  $\pi$  is closed.  $\underline{v}_m$  and  $\bar{v}_m$  are the upper and lower bounds on the voltage magnitude at bus  $m$ . To obtain a tractable MILP  
155 formulation, we use the linearized DistFlow approximation of the power flow equations [23], as in other related research, e.g., [11, 12, 24]. Other linear approximations would also provide tractable MILP formulations. We note that this research area would benefit from further studies that rigorously assess the  
160 accuracy of the Linearized DistFlow and other approximations during extreme events. This is an important topic for future work.

Based on the linearized DistFlow equations, the voltages of the buses are related as in (4a), where the big-M method is used to decouple the voltages of two disconnected buses to account for the behavior of switches. Furthermore, the voltages of the buses should be within the allowable range as dictated in (4b).

$$(1 - \varsigma_{\pi,t,m,n})M - \frac{r_{m,n} \cdot p_{\pi,t,m,n}^E + x_{m,n} \cdot q_{\pi,t,m,n}^E}{v_1} \leq v_{\pi,t,n} - v_{\pi,t,m} \quad (4a)$$

$$\leq (\varsigma_{\pi,t,m,n} - 1)M - \frac{r_{m,n} \cdot p_{\pi,t,m,n}^E + x_{m,n} \cdot q_{\pi,t,m,n}^E}{v_t}$$

$$\underline{v}_m \leq v_{\pi,t,m} \leq \bar{v}_m \quad (4b)$$

We linearize the line flow limits specified in terms of apparent power as shown in (5), where  $\bar{S}_{m,n}$  is the capacity of line  $(m, n)$ .

$$- \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^E \leq \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (5a)$$

$$- \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq q_{\pi,t,m,n}^E \leq \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (5b)$$

$$- \sqrt{2}\varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^E + q_{\pi,t,m,n}^E \leq \sqrt{2}\varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (5c)$$

$$- \sqrt{2}\varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^E - q_{\pi,t,m,n}^E \leq \sqrt{2}\varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (5d)$$

Let  $\alpha_{\pi,t,m}$  be a binary variable indicating whether the distributed generator at bus  $m$  is generating power during time  $t$  in scenario  $\pi$ .  $\bar{P}_m^{DG}$ ,  $\bar{Q}_m^{DG}$ ,  $\underline{P}_m^{DG}$ ,



and  $\underline{Q}_m^{DG}$  are the maximum/minimum active/reactive power outputs of the distributed generator at bus  $m$ . Constraint (6) ensures that the active and reactive power outputs of distributed generators are within their allowed ranges.

$$\alpha_{\pi,t,m} \cdot \underline{P}_m^{DG} \leq P_{\pi,t,m}^{DG} \leq \alpha_{\pi,t,m} \cdot \overline{P}_m^{DG} \quad (6a)$$

$$\alpha_{\pi,t,m} \cdot \underline{Q}_m^{DG} \leq Q_{\pi,t,m}^{DG} \leq \alpha_{\pi,t,m} \cdot \overline{Q}_m^{DG} \quad (6b)$$

Finally, (7) enforces a radial configuration of distribution system by using the spanning tree approach wherein each bus except the root bus has either one or zero parent buses [25]. In (7),  $\lambda_{\pi,t,m,n}$  is a binary variable indicating whether bus  $n$  is the parent of bus  $m$  at time  $t$  in scenario  $\pi$  and  $\Psi_m$  denotes the set of buses connected to bus  $m$  by a line.

$$\lambda_{\pi,t,m,n} + \lambda_{\pi,t,n,m} = \varsigma_{\pi,t,m,n}, \quad \forall n \in \Psi_m \quad (7a)$$

$$\sum_{m \in \Psi_m} \lambda_{\pi,t,m,n} \leq 1, \quad \forall n \in \Psi_m \quad (7b)$$

$$\lambda_{\pi,t,1,n} = 0, \quad \forall n \in \Psi_{\text{root}} \quad (7c)$$

## 2.2. Mobile Generators

As shown in Fig. 1, mobile generators can be dispatched from depots to distribution buses to supply loads after HILP events. This motivates careful consideration of travel and setup time requirements. In this regard, let  $\chi$  and  $\Xi$  denote the sets of depots and mobile generators, respectively.  $\delta_{\pi,dp,mg,m}$  is a binary variable indicating whether mobile generator  $mg$  moves from depot  $dp$  to bus  $m$  in scenario  $\pi$ .  $\beta_m$  is the maximum number of mobile generators that can be installed at bus  $m$ . Constraints (8)–(16) model mobile generator scheduling after the occurrence of a major disaster. Constraint (8) indicates that at most  $\beta_m$  mobile generators can be dispatched to bus  $m$ .

$$\sum_{mg \in \Xi} \sum_{dp \in \chi} \delta_{\pi,dp,mg,m} \leq \beta_m \quad (8)$$

Constraint (9) indicates that each mobile generator cannot be dispatched more than once.

$$\sum_{m \in \mathcal{N}} \sum_{dp \in \mathcal{X}} \delta_{\pi, dp, mg, m} \leq 1 \quad (9)$$

Let  $\Gamma_{\pi, dp, mg, m}$  denote the arrival time of mobile generator  $mg$  from depot  $dp$  at bus  $m$  in scenario  $\pi$ . Let  $T_{dp, m}$  denote the time needed for a mobile generator to connect to bus  $m$  when departing from depot  $dp$ .  $\gamma_{\pi, t, mg, m}$  is a binary variable indicating whether a mobile generator arrives at bus  $m$  at time  $t$  in scenario  $\pi$ . Constraint (10) models the total time required to connect mobile generators to the distribution buses. Constraints (11) and (12) then ensure consistency among the decision variables related to the connection times. For example, if the arrival time of mobile generator 1 to bus 14 is 20 minutes (i.e.,  $\Gamma_{\pi, dp, 1, 14} = 20$ ), the binary variable  $\gamma_{\pi, t, 1, 14}$  will be one for  $t = 20$  and zero for  $t \neq 20$ .

$$\Gamma_{\pi, dp, mg, m} = T_{dp, m} \times \delta_{\pi, dp, mg, m} \quad (10)$$

$$\sum_{t \in \mathcal{T}} t \times \gamma_{\pi, t, mg, m} \geq \sum_{dp \in \mathcal{X}} \Gamma_{\pi, dp, mg, m} \quad (11)$$

$$\sum_{t \in \mathcal{T}} t \times \gamma_{\pi, t, mg, m} \leq \sum_{dp \in \mathcal{X}} \Gamma_{\pi, dp, mg, m} + 1 - \varepsilon \quad (12)$$

$\kappa_{\pi, t, mg, m}$  is a binary variable indicating whether mobile generator  $mg$  is connected to bus  $m$  at time  $t$  in scenario  $\pi$ . Equation (13) couples  $\gamma_{\pi, t, mg, m}$  and  $\delta_{\pi, dp, mg, m}$ .

$$\sum_{t \in \mathcal{T}} \gamma_{\pi, t, mg, m} = \sum_{dp \in \mathcal{X}} \delta_{\pi, dp, mg, m} \quad (13)$$

In our model, we consider a time frame during which mobile generators can move from their depot and connect to one bus but cannot subsequently disconnect and move to another bus. This is enforced by (14).

$$\kappa_{\pi, t, mg, m} = \sum_{t=1}^t \gamma_{\pi, t, mg, m} \quad (14)$$

$\bar{P}_{mg}$  and  $\bar{Q}_{mg}$  are the maximum active and reactive power outputs of mobile generator  $mg$ , as modeled by (15) and (16).

$$0 \leq P_{\pi,t,m}^{MG} \leq \sum_{mg \in \Xi} \kappa_{\pi,t,mg,m} \times \bar{P}_{mg} \quad (15)$$

$$0 \leq Q_{\pi,t,m}^{MG} \leq \sum_{mg \in \Xi} \kappa_{\pi,t,mg,m} \times \bar{Q}_{mg} \quad (16)$$

165 *2.3. Underground Lines*

Construction of underground lines can significantly increase the resilience of the power supplied to critical loads after HILP events, especially when installed while considering the capabilities of with mobile generators. In this regard, let  $\tau_{m,n}$  be a binary variable indicating whether an underground line is constructed  
 170 between buses  $m$  and  $n$ . The voltages of the underground lines' terminal buses are related by (17a). Constraint (17b) states that underground lines are only in a serviceable state if the lines are constructed (i.e.,  $\tau_{m,n} = 1$ ) and the lines' switches are closed (i.e.,  $\varsigma_{\pi,t,m,n} = 1$ ).

$$\begin{aligned} (1 - \varsigma_{\pi,t,m,n}) M - \frac{r_{m,n} \cdot p_{\pi,t,m,n}^A + x_{m,n} \cdot q_{\pi,t,m,n}^A}{v_1} \\ \leq v_{\pi,t,n} - v_{\pi,t,m} \\ \leq (\varsigma_{\pi,t,m,n} - 1) M - \frac{r_{m,n} \cdot p_{\pi,t,m,n}^A + x_{m,n} \cdot q_{\pi,t,m,n}^A}{v_1} \end{aligned} \quad (17a)$$

$$\tau_{m,n} \geq \varsigma_{\pi,t,m,n} \quad (17b)$$

The linearized form of the underground lines' flow limits is modeled in (18).

$$- \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^A \leq \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (18a)$$

$$- \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq q_{\pi,t,m,n}^A \leq \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (18b)$$

$$- \sqrt{2} \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^A + q_{\pi,t,m,n}^A \leq \sqrt{2} \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (18c)$$

$$- \sqrt{2} \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \leq p_{\pi,t,m,n}^A - q_{\pi,t,m,n}^A \leq \sqrt{2} \varsigma_{\pi,t,m,n} \cdot \bar{S}_{m,n} \quad (18d)$$

Let  $\ell_{mn}$  denote the length of the line between buses  $m$  and  $n$ .  $C^A$ ,  $C^{RCS}$ , and  $C^{\text{invest}}$  are the per-length cost of constructing underground lines, the cost of installing remote-controlled switches, and the total budget of the distribution

Table 1: Computational Scaling of the Stochastic MILP Model

Num. of	Computational scaling	IEEE 123-Bus
Binary variables	$ \Pi  T (2 L +3 N )$ $+ \Pi  \Xi  N + N ^2+ \eta $	324,554
Continuous variables	$2 \Pi  T ( N + L + \eta + DG )$ $+3 \Pi  N  \Xi $	279,780
Constraints	$ T  \Pi (7 N +5 L +3 \Xi  N +7 \eta )$ $+2 DG +2 L +1+2 N )$ $+2 \Pi  \Xi +4 \Pi  N  \Xi + \eta +2$	1,908,447

utility, respectively.  $\eta^A$  is the maximum allowed number of underground lines. Constraint (19a) ensures that the total investment cost of underground lines and remote-controlled switches is no greater than the total budget of the distribution utility. Also, (19b) states that the number of constructed underground lines must be within the specified limit.

$$\sum_{(m,n) \in \mathcal{L}^A} \left( \frac{1}{2} \ell_{mn}^A \cdot C^A + C^{\text{RCS}} \right) \cdot \tau_{m,n} \leq C^{\text{invest}} \quad (19a)$$

$$\sum_{(m,n) \in \mathcal{L}^A} \tau_{m,n} / 2 \leq \eta^A \quad (19b)$$

#### 2.4. Final Model

The final model is summarized as follows:

$$\min (1) \text{ s.t. } (2) - (19) \quad (20)$$

175 Table 1 describes the computational scaling of the proposed model. Ob-  
serve that the number of variables has a linear relationship with the number of  
time periods and the number of scenarios. In this paper, the objective func-  
tion minimizes the load curtailment in the aftermath of extreme events while  
constraining the number of undergrounded lines with budget and/or cardinality  
180 constraints. The ability to control the parameters on the right hand sides of  
these constraints (i.e., the total budget and/or number of undergrounded lines)

provides the user of the proposed model with the ability to control the tradeoff between infrastructure cost and the additional resilience achieved.

### 3. Results and Discussion

185 In this section, the model from Section 2 is applied to the IEEE 123-bus test system to demonstrate the effectiveness of the proposed approach.

#### 3.1. IEEE 123-Bus Test System

This system is operated at a nominal voltage of 4.16 kV with total active and reactive power demand of 3.49 MW and 1.92 MVar, respectively. The modified  
190 test system is shown in Fig. 2. Three distributed generators are installed in the test system with the parameters given in Table 2. Also, five mobile generators are available at the depot location with the parameters shown in Table 3. Here, 14 of the loads are classified as “critical” as shown by the triangles in Fig. 2. The underground distribution lines are equipped with remote-controlled  
195 switches. The costs of constructing underground distribution lines and installing remote-controlled switches are given in Table 4. Note that the possibility of existing underground lines can be considered in the developed model by assigning zero failure probability to the existing underground distribution lines in the scenario generation algorithm in the case of certain events. For other events,  
200 such as landslides, earthquakes, and flooding, corresponding probabilities can be modeled.

We solved the stochastic MILP problem described in Section 2 to a 1% optimality gap using the Gurobi 9.5.0 solver on a computing node of the Partnership for an Advanced Computing Environment (PACE) cluster at the Georgia Institute of Technology [26]. This computing node has a 24-core CPU and 32 GB of  
205 RAM. We implement the model in GAMS 38.1 [27].

#### 3.2. Simulation Results

We next describe the results obtained from solving the MILP model (20) to determine locations for adding new underground lines while jointly model-

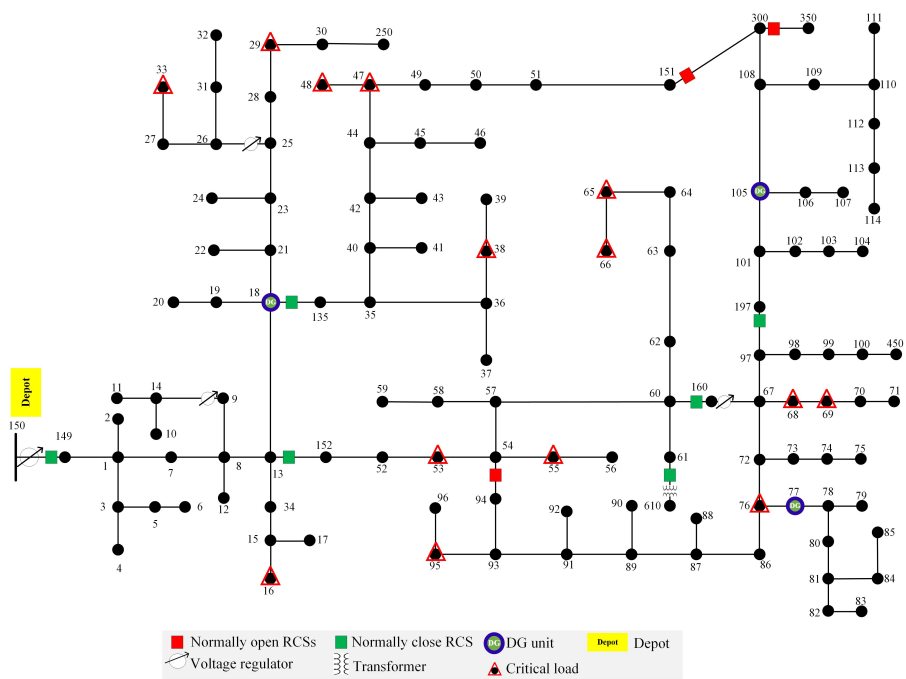


Figure 2: Modified IEEE 123-bus test system.

Table 2: Parameters of Installed Distributed Generators in the IEEE 123-Bus Test System

Distributed Generator	Bus	$\bar{P}_m^{DG}$	$\underline{P}_m^{DG}$	$\bar{Q}_m^{DG}$	$\underline{Q}_m^{DG}$
DG1	18	400 kW	0 kW	300 kVAr	-300 kVAr
DG2	77	700 kW	0 kW	600 kVAr	-600 kVAr
DG3	105	700 kW	0 kW	600 kVAr	-600 kVAr

Table 3: Parameters of Available Mobile Generators

Mobile Generators	$\bar{P}_{MG}$	$\bar{Q}_{MG}$
MG1	200 kW	150 kVAr
MG2	300 kW	200 kVAr
MG3	500 kW	400 kVAr
MG4	650 kW	550 kVAr
MG5	700 kW	600 kVAr

Table 4: Investment Costs

Investment	Cost
Underground line construction	\$1M/mile [28]
Remote-controlled switches	\$15k [5]

210 ing many disaster scenarios. For each scenario, this model considers the optimal dispatch of the mobile generators when determining where to add new underground lines. The results show that this holistic modeling approach in the planning phase improves the system’s restoration performance during the operational phase.

215 *3.2.1. Planning*

To find the optimal locations for constructing new underground distribution lines, we solve the proposed MILP model (20) with 20 scenarios. To capture the uncertainties associated with damage to distribution lines, we first generated 1000 damage scenarios by a Monte Carlo sampling technique along with fragility curves [24] and then reduced these to 20 scenarios using GAMS’ `scenred` tool.

220

The optimal locations for varying numbers of underground lines, their associated costs, and computational times are given in Table 5. As an example, with a \$1 million budget, the solution constructs five underground lines with a total length of 4100 feet (1250 meters) to link certain critical loads such that the operator can simultaneously re-energize them after a disaster. In combination with the two available normally open switchable lines (i.e., lines 54-94 and 151-300), the IEEE 123-bus test system will have total seven switchable lines, which significantly improves the reconfiguration capability of the system. Consider, for instance, a scenario where buses 29 and 47 are isolated due to a natural disaster. In this situation, the system operator can restore both loads simultaneously by dispatching one mobile generator to the location of the nearest bus instead of dispatching two mobile generators separately to the locations of buses 29 and 47.

### 3.2.2. Operation

As a representative example, we next focus on the case in Table 5 where six underground distribution lines are constructed; see the green lines marked with a star in Fig. 3 for their locations. Note that the distribution system is operated radially by appropriately configuring the switches on the newly added underground lines. For a representative scenario where 27 of the lines have been damaged due to a natural disaster (marked with red lightning bolts in Fig. 3), we use the proposed model (20) with fixed locations for underground lines and this particular damage scenario to optimize the dispatch of mobile generators and the system configuration via the switch statuses in order to restore service to the maximum extent possible. In this paper, the faults caused by extreme events are considered to be open-circuit. Fig. 3 visualizes the solution, including the dispatch of available mobile generators, and Table 6 provides the optimal dispatch of mobile generators and their associated arrival times. We observe that the simultaneous utilization of mobile generators and reliable links between critical loads leads to wide-area energization of the distribution system after the disaster. The mobile generators are directly connected to critical loads and also



Table 5: Optimal Distribution Line Undergrounding

Num.	Underground Lines	Length (ft)	Cost	Time (sec)
6	16-95, 53-95, 29-47, 33-48, 38-65, 69-76	4900	\$1,108k	1799
5	16-95, 29-47, 33-48, 38-65, 69-76	4100	\$926.5k	1879
4	29-47, 33-48, 38-65, 69-76	3100	\$707.1k	2233
3	29-47, 33-48, 69-76	2200	\$506.7k	2492
2	29-47, 38-65	1500	\$344.1k	1062
1	29-47	700	\$162.6k	334

supply adjacent high-priority customers either through available overhead lines or the newly constructed underground lines. Therefore, the model dispatches the mobile generators to restore the critical loads as fast as possible. For instance, mobile generator 1 is dispatched to both energize the critical load at bus 16 and simultaneously serve the critical load at bus 95. Mobile generator 2 energizes the critical load at buses 53 and 55 after 30 minutes; and, it also energizes the adjacent interrupted loads through the line 54-94. Likewise, mobile generator 3 restores the critical loads at buses 38, 65, and 66 by using the constructed underground line 38-65. After being connected to bus 33, mobile generator 4 also re-energizes the neighboring critical loads such as loads at buses 29, 47, and 48. Finally, critical loads at buses 68, 69, and 76 are restored within 55 minutes by mobile generator 5. Accordingly, simultaneous utilization of constructed underground lines and mobile generators alongside the surviving lines leads to fast re-energization of critical loads.

Fig. 4 shows a temporal representation of the service restoration process, including the total supplied load, the supplied critical load, and the power outputs of the mobile generators. As more mobile generators reach their destinations and connect to the system, the total supplied load increases from 19.5% to 82.1% at the end of the restoration process. Accordingly, mobile generators can

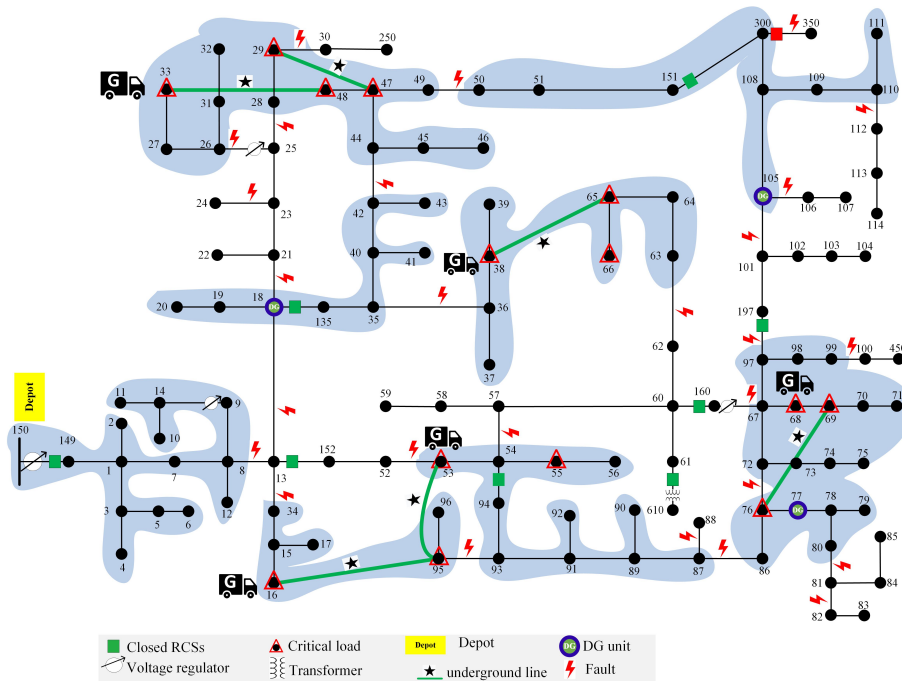


Figure 3: Optimal dispatch of mobile generators in the IEEE 123-bus test system.

270 significantly boost the resilience level of the distribution system owing to their  
movable nature, which provides the system operator with considerable flexibil-  
ity to respond to extreme events. Moreover, as shown in by the utilization rates  
(ratio of power output to generation capacity) in Table 7, the solution effect-  
ively uses the capabilities of the mobile generators. Observe that the proposed  
275 model maximizes the utilization rate of mobile generators by dispatching them  
to proper load points and taking the advantage of constructed underground dis-  
tribution lines which can energize some of their neighboring critical loads such  
as loads at buses 29, 47, 48, 65, 76, and 95. The solution energizes all critical  
loads within the first 40 minutes of the restoration process

280 Next, to verify the effect of tie-lines in the resilience level of distribution sys-  
tems, the re-energization of the system in the aftermath of the extreme event  
is evaluated for different investment levels for underground line construction in  
the system. In doing so, Fig. 5 plots the load supplied during the restoration

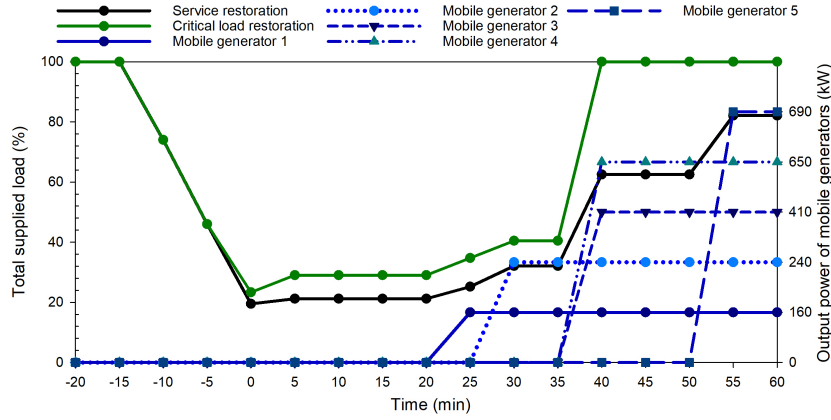


Figure 4: Optimal service restoration in the IEEE 123-bus test system.

processes corresponding to investment plans with differing numbers of under-  
 285 ground lines. Also, the tradeoff between infrastructure cost and the additional  
 resilience achieved is shown in Table 8. When six new lines are constructed,  
 the total energy supplied within the first two hours increases by 3.985 MWh  
 (57.1%) compared to the case without any new lines. As a result, increasing the  
 number of constructed underground lines helps the simultaneous energization  
 290 of critical loads and significantly speeds up the restoration process. The results  
 indicate that the presence of underground distribution lines also increases the  
 utilization rate of mobile generators by transmitting some of their output power  
 to the neighboring critical loads.

To assess the value of additional mobile generators, Fig. 6 visualizes the load  
 295 served during restoration processes that use various numbers of mobile gener-  
 ators. For varying numbers of available mobile generators in the system, the total  
 load supplied increases by 1.7%, 7.4%, 16%, 30.1%, 49%, and 62.6% for zero,  
 one, two, three, four, and five available mobile generators, respectively. Accord-  
 300 ingly, mobile generators are valuable assets in restoring the interrupted loads  
 during extreme events. Note that the presence of constructed underground lines  
 also adds to the value of additional mobile generators by allowing simultaneous  
 restoration of several outaged areas.

Table 6: Optimal Dispatch of Mobile Generators

Mobile Generator	Bus	Arrival Time (min)
MG1	16	25
MG2	53	30
MG3	38	40
MG4	33	40
MG5	68	55

Table 7: Utilization Rate of Mobile Generators

Mobile Generators Utilization Rate			
Mobile Generator	Coordinated case	Uncoordinated case	Heuristic case
MG1	80.0%	40.0%	50.0%
MG2	80.0%	100%	100%
MG3	82.0%	68.0%	99.0%
MG4	100%	78.5%	24.6%
MG5	98.6%	82.1%	34.4%
Total	91.4%	76.8%	55.1%

Table 8: Resilience level achieved for varying number of underground lines

Number of underground lines	Cost	Supplied load
6	\$1,108k	82.1%
5	\$926.5k	82.1%
4	\$707.1k	80.9%
3	\$506.7k	78.7%
2	\$344.1k	76.5 %
1	\$162.6k	73.6%

Finally, to verify the merits of the proposed model in optimizing the location of underground lines in the planing phase, we compare the proposed mobile generator restoration process aware planning model with two alternatives to our proposed approach. In the context of our proposed model, we first show that

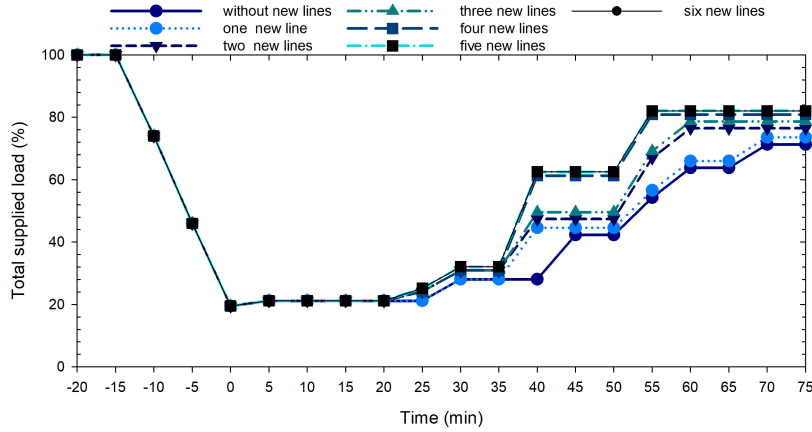


Figure 5: Load supplied with and without underground lines.

long-term line undergrounding decisions which are cognizant of short-term mobile generator deployments yield superior results relative to undergrounding decisions made without considering mobile generators. To this end, we consider an alternative “uncoordinated” approach which sets the number of mobile generators to zero in the stochastic MILP model and computes the optimal locations for six new underground lines (within the budget constraints) for the specified damage scenarios. Then, by fixing the locations of these underground lines, the model is executed with five mobile generators for the same damage scenario as analyzed previously. To compare the resilience level of the distribution system for coordinated and uncoordinated strategies, the restoration process for the total load and the critical load for two strategies are depicted in Fig. 7. Observe that the total energy supplied to critical loads in the uncoordinated case decreases by 12.53% compared to the coordinated case in Fig. 4. Also, the final amounts of total load supplied (71.06%) and critical load supplied (92.42%) in the uncoordinated case are less than the corresponding values (82.1% and 100%, respectively) for the coordinated case in Fig. 4. Moreover, as shown in Table 7, the mobile generators in the coordinated case have a higher overall utilization rate compared to the uncoordinated case (90.67% versus 76.8%). In addition, we also consider an alternative heuristic approach that undergrounds

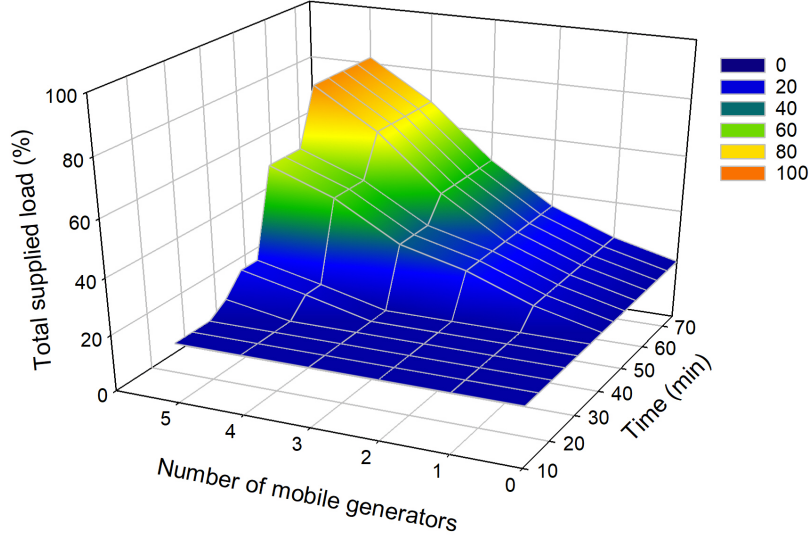


Figure 6: Load supplied with various numbers of mobile generators.

lines between the closest critical loads in the planning phase (within the budget constraints). This heuristic approach does not use an optimization model.

The restoration processes for the total load and the critical load for each approach are visualized in Fig. 7. Furthermore, the utilization rate of mobile generators are shown in Table 7. Table 9 summarizes comparisons among the coordinated, uncoordinated, and heuristic planning approaches. These comparisons show that considering line undergrounding and mobile generators simultaneously in the proposed model results in better service restoration performance in the aftermath of a natural disaster. For instance, the proposed (coordinated) approach serves over 11% and 23% and more total load than the uncoordinated and heuristic approaches, respectively, with similar levels of improvements in the critical load supplied and the mobile generator utilization rate.

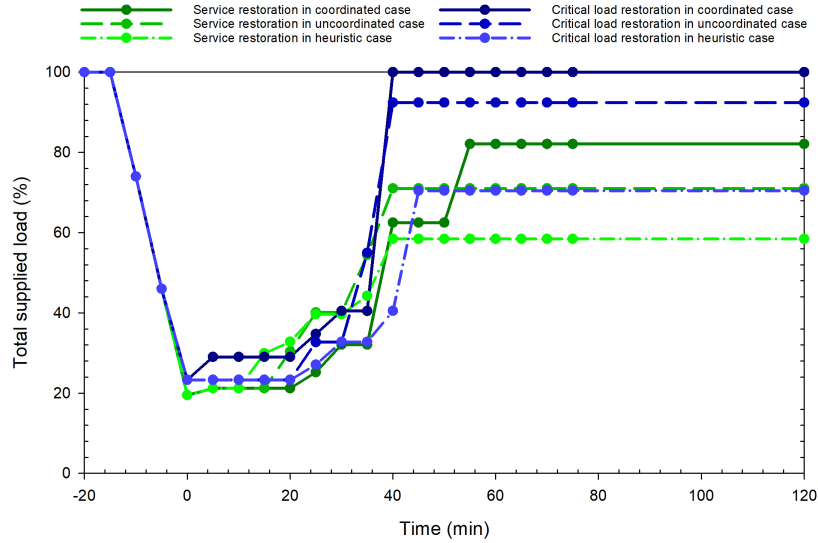


Figure 7: Load supplied when undergrounding decisions are made via considering mobile generators (coordinated approach), not considering mobile generators (uncoordinated approach), and a heuristic approach.

#### 4. Conclusion

To provide critical loads with a highly reliable power supply, this paper  
 340 proposes a stochastic MILP model that considers both the deployment of mobile  
 generators and the optimal locations to underground lines in order to provide  
 utilities with 1) flexible and reliable power sources and 2) resilient connections  
 between locations with critical loads. The proposed model has been evaluated  
 using a standard test system and the simulation results confirm the effectiveness  
 345 of these measures in coping with extreme events.

In future work, we plan to use a more realistic transportation system model  
 for mobile generators that includes, for instance, the potential for traffic inter-  
 ruptions. Also, extending the developed model by including the repair process  
 of damaged components is another direction for the future work. Finally, an-  
 350 other direction for extending the work is to consider a nonlinear representation  
 of the power flow equations, ideally a three-phase unbalanced power flow model,

Table 9: Comparison of coordinated, uncoordinated and heuristic planning strategies

Comparison method	Planning Strategy		
	Coordinated	Uncoordinated	Heuristic
Final value of total load supplied	82.10%	71.06%	58.45%
Final value of critical load supplied	100%	92.42%	70.50%
Total mobile generator utilization rate	91.40%	76.80%	55.10%

to more accurately represent the behavior of heavily stressed systems.

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