A Multiperiod Optimal Power Flow Approach to Improve Power System Voltage Stability Using Demand Response

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Abstract—The increasing penetration of renewables has driven power systems to operate closer to their stability boundaries, increasing the risk of instability. We propose a multiperiod optimal power flow approach that uses demand responsive loads to improve steady-state voltage stability, which is measured by the smallest singular value (SSV) of the power flow Jacobian matrix. In contrast to past work that employs load shedding to improve stability, our approach improves the SSV by decreasing and increasing individual loads while keeping the total loading constant to avoid fluctuation of the system frequency. Additionally, an energy payback period maintains the total energy consumption of each load at its nominal value. The objective function balances SSV improvements against generation costs in the energy payback period. We develop an iterative linear programming algorithm using singular value sensitivities to obtain an AC-feasible solution. We demonstrate its performance on two IEEE test systems. The results show that demand response actions can improve static voltage stability, in some cases more cost effectively than generation actions. We also compare our algorithm’s performance to that of an iterative nonlinear programming algorithm from the literature. We find that our approach is approximately 6 times faster when applied to the IEEE 118-bus system, allowing us to demonstrate its performance on the IEEE 118-bus system.

Index Terms—Demand response, optimal power flow, voltage stability, singular value, iterative linearization.

NOTATION

Functions
- \( C(\cdot) \) Total generation cost
- \( P_{n}^{Re}(\cdot) \) Real power injection at bus \( n \)
- \( P_{n}^{Q}(\cdot) \) Reactive power injection at bus \( n \)
- \( H_{nm}(\cdot) \) Line flow for line \((n,m)\)
- \( F_{P_{nm}}(\cdot) \) Linearization of \( P_{nm} \)
- \( F_{Q_{nm}}(\cdot) \) Linearization of \( Q_{nm} \)
- \( \mathbf{H}_{nm}(\cdot) \) Linearization of \( H_{nm} \)

Sets
- \( \mathcal{N} \) Set of all buses
- \( \mathcal{S}_{PV} \) Set of all PV buses
- \( \mathcal{S}_{PQ} \) Set of all PQ buses
- \( \mathcal{S}_{G} \) Set of buses with generators
- \( \mathcal{S}_{DR} \) Set of buses with responsive loads
- \( T \) Set of time periods within optimization problem

Variables & Parameters
- \( J \) Jacobian matrix
- \( P_{d,n} \) Real power demand at bus \( n \)
- \( P_{g,n} \) Real power generation at bus \( n \)
- \( P_{loss} \) Total power loss in the system
- \( Q_{d,n} \) Reactive power demand at bus \( n \)
- \( Q_{g,n} \) Reactive power generation at bus \( n \)
- \( T_{t} \) Length of time period \( t \)
- \( u \) Left singular vector
- \( V_{n} \) Voltage magnitude at bus \( n \)
- \( w \) Right singular vector
- \( \alpha \) Weighting factor
- \( \epsilon \) Loss management strategy parameter
- \( \theta_{n} \) Voltage angle at bus \( n \)
- \( \lambda \) Eigenvalue of a matrix
- \( \mu_{n} \) Ratio between reactive and real demand at bus \( n \)
- \( \sigma \) Singular value of a matrix
- \( \sigma_{0} \) Smallest singular value of a matrix
- \( \Sigma, U, W \) Singular Value Decomposition (SVD) matrices
- \( \chi \) Operating point

Bold symbols denote vectors including all variables of a type. Overlines and underlines represent the upper and lower limits for variables. Numbers in the parentheses (\( ' \)) refer to the period number. Subscript ‘ref’ denotes the slack bus. Superscript ‘\( t \)’ denotes the current value of a variable and superscript ‘\( T \)’ denotes the transpose of a matrix. The notation \( X \succeq 0 \) means that \( X \) is a positive semidefinite matrix. For notational simplicity, we assume that each bus has at most one generator and at most one load.

I. INTRODUCTION

Increasing penetrations of renewable energy sources can negatively impact power systems stability [1], [2]. Specifically, power-electronics-connected fluctuating renewable generation from wind and solar introduce more variability in operating points, reduce system inertia, and decrease the controllability of active power injections. A variety of methods have been proposed to improve power system stability margins including generation dispatch [3], locating/sizing distributed generation [4], and use of advanced power electronic devices [5]. Demand Response (DR) can also be used to improve power system stability. For example, [6]–[8] propose methods to coordinate loads to help balance supply and demand, improving frequency stability. As we increase the controllability of distributed electric loads to enable their participation in...
a variety of DR programs and electricity markets [9], we also unleash their potential to provide a variety of stability-related services not typically rewarded in existing programs or markets. However, harnessing loads for this purpose requires the development of new algorithms.

In this paper, we propose a multi-period optimal power flow (OPF) approach that uses DR to improve steady-state voltage stability. In contrast to past work that developed load shedding approaches to improve voltage stability [10], [11], we decrease and increase loads while keeping the total loading constant to avoid fluctuation of the system frequency, and we “pay back” the changes to each load so its total energy consumption is unchanged. We envision that such an approach would be used only occasionally, when voltage stability margins are below those desired, but not so small that emergency actions are immediately necessary. DR actions could be executed quickly while ramp-rate-limited generators begin to respond, eventually relieving the loads. Beyond developing the problem formulation and solution algorithm, our objective is to compare the stability margin improvement and cost of load actions to those of generator actions in order to understand both the advantages and disadvantages of the approach.

Steady-state voltage stability margins estimate how far an operating point is from instability due to voltage collapse. Various margins have been proposed, including the loading margin, which is the distance between the current operating point and the maximum loading point [3]. The loading margin is calculated using continuation power flow methods, where the load and generation are usually increased uniformly (in a multiplicative sense) throughout the system [12], [13]. A drawback of this method is that it assumes a single direction of changes in power injections. Another margin is the distance to the closest saddle-node bifurcation (SNB) point. References [14], [15] derive a system of nonlinear equations from which we can calculate the optimal control direction, which is antiparallel to the normal vector at the closest SNB. However, changing the loads in this direction may change the total loading. A third margin, which we use in this paper, is the smallest singular value (SSV) of the power flow Jacobian matrix [16]–[20].

The SSV gives us a measure of how close the Jacobian is to being singular, i.e., power flow infeasibility. Feasibility and stability are closely linked [21]. The advantages of using the SSV as a voltage stability margin are that 1) it captures any direction of changes in power injections and 2) there exist approximate mathematical formulations suitable for inclusion in optimization problems, e.g., [22], [23]. The disadvantages of using the SSV are that 1) it only provides implicit information on the distance to the solvability boundary, 2) it does not capture the impact of all engineering constraints (e.g., reactive power limits) could be reached prior to power flow singularity [24], and 3) it may not be well-behaved, specifically, [25] found that the SSV at voltage collapse varies significantly as function of the loading direction (see Fig. 3 of [25]). Additionally, 4) its numerical value is system-dependent [19] and so the threshold value for a particular system would need to be determined from operator experience. Moreover, 5) the nonlinear programming (NLP) algorithm for solving approximate mathematical formulation [22] does not scale to realistically-sized system. Despite these issues, we base our approach on the SSV in order to exploit the approximate mathematical formulation [22], [23] and we develop an improved solution algorithm that scales significantly better. Recognizing the potential advantages of other stability margins, our ongoing work is exploring the development of analogous formulations based on other stability margins.

The contributions of this paper are four-fold. 1) We formulate the multi-period optimal power flow problem. In the first period, we maximize the SSV by changing the loading pattern subject to the AC power flow equations, engineering limits, and a constraint that forces the total loading to be constant. The second period minimizes the generation cost while paying back energy to each load and maintaining the SSV. 2) We develop a computationally-tractable iterative linear programming (LP) solution algorithm using singular value sensitivities [11], [26] and benchmark it against the NLP algorithm in [22]. 3) We conduct case studies using the IEEE 9- and 118-bus systems to determine optimal loading patterns and assess algorithmic performance. 4) We compare between the cost/performance of DR versus generation in improving the voltage stability.

This paper builds on our preliminary work [27], which developed a single-period formulation that uses DR to maximize the SSV, but does not consider energy payback. In [27], we proposed an iterative LP solution algorithm using eigenvalue sensitivities; however, our new algorithm in this paper is far more computationally tractable, allowing us to demonstrate scalability to the IEEE 118-bus system.

The remainder of the paper is organized as follows. Section II describes the problem and our assumptions. Section III presents the formulation and solution algorithm. Section IV shows the results of our case studies and Section V concludes.

II. Problem Description & Assumptions

A conceptual illustration of the problem is shown in Fig. 1. The system is initially operating with an adequate stability margin at an operating point (star) determined via unit commitment and economic dispatch. A disturbance happens (e.g., a line or generator goes out of service) causing the operating point to move towards the feasibility/stability boundary (i.e., to Operating Point 0) and the stability margin to drop. At this time, the system is prone to instability because slight variations in power injections might cause the operating point to leave the stable operating region. The system operator dispatches quick-acting resources including DR to maximize the stability margin (Operating Point 1). After a short period of time, the system operator determines the minimum cost dispatch of slower-acting generators that relieves the loads, pays back the changes made to each load at Operating Point 1, and maintains/improves the stability margin (Operating Point 2). The payback sets the energy consumed by each load while at Operating Point 2 to its nominal (i.e., baseline) consumption plus/minus the energy not consumed/consumed while at Operating Point 1. As shown in Fig. 1(b), at Operating Point 2, the achievable stability margin is a function of whether or not the
Beyond our base case, we also investigate cases in which the length of Period 2. Lengths $T_1$ and $T_2$ are not necessarily equal, as shown in Fig. 2. For notational simplicity, we assume the real power demand at bus $n$, $P_{d,n}(t)$, is constant within a time period and the nominal real power demand in all periods is equal to $P_{d,n}(0)$; however, the formulation could be easily extended to incorporate time-varying demands.

Let $\mathcal{N}$ be the set of all buses, $\mathcal{S}_{PV}$ be the set of all PV buses, and $\mathcal{S}_{Q}$ be the set of all PQ buses. Additionally, let $\mathcal{S}_{G}$ be the set of all buses with generators, i.e., all PV buses in addition to the slack bus, and let $\mathcal{S}_{DR}$ be the set of buses with responsive loads; the buses comprising $\mathcal{S}_{DR}$ may be PV or PQ buses. In our case studies, we assume that a portion of the existing loads in the network are responsive.

The multiperiod optimal power flow problem determines the operating points in each time period that balance the two objectives: maximizing the SSV of the power flow Jacobian matrix in Period 1 and minimizing the generation cost in Period 2. The general formulation is as follows.

$$\min \left\{ P_{\text{e}}(t), Q_{\text{e}}(t), P_{a}(t), Q_{a}(t), V(t), \theta(t), \sigma(t) \right\}$$

$$\text{s.t.} \quad \forall t \in T$$

$$\sigma(t) = \sigma_{\min}\{J(\theta(t), V(t))\}$$

$$\mathcal{F}^I_n(\theta(t), V(t)) = P_{g,n}(t) - P_{d,n}(t) \quad \forall n \in \mathcal{N}$$

$$\mathcal{F}^Q_n(\theta(t), V(t)) = Q_{g,n}(t) - Q_{d,n}(t) \quad \forall n \in \mathcal{N}$$

$$\sum_{n \in S_{\text{PV}}} P_{d,n}(1) = \sum_{n \in S_{\text{DR}}} P_{d,n}(0) + \varepsilon (P_{\text{loss}}(0) - P_{\text{loss}}(1))$$
In Period 2, generator real power generation and voltage magnitudes are allowed to change within their limits, specifically,

\[ P_{g,n}(2) \leq P_{g,n}(2) \leq \bar{P}_{g,n}(2) \quad \forall n \in S_G \]
\[ V_n(2) \leq V_n(2) \leq \bar{V}_n(2) \quad \forall n \in N \]

We investigate seven additional cases in which we vary the decision variables that are allowed to change in Period 1 (specifically, \( P_{g,ref}, P_{g,n} \forall n \in S_{PV}, V_n \forall n \in S_G, \) and \( P_{d,n}, Q_{d,n} \forall n \in S_{DR} \)), the loss management strategy, and, for cases in which generator real power generation is allowed to change in Period 1, whether or not we impose a ramp rate. The cases and associated results, which will be discussed later, are summarized in Table III.

The difficulty in solving (1) stems from the existence of the implicit constraint (1b). Because the singular values of a matrix \( A \) are the square roots of the eigenvalues of \( A^T A \), we can replace (1b) with

\[ J(t)^T J(t) - \lambda_0(t) I \succeq 0 \]
\[ \sigma_0(t) = \sqrt{\lambda_0(t)} \]

where the semidefinite constraint (2) forces \( \lambda_0 \) to be the smallest eigenvalue of \( J(t)^T J(t) \). \( I \) is an identity matrix of appropriate size, and we have simplified the expression for the power flow Jacobian matrix for clarity. The SSV of \( J(t) \) is the square root of \( \lambda_0 \), as shown in (3).

### B. Existing Solution Approaches

A variety of methods have been used to solve problems similar to (1). For example, [30] computes the Hessian of (1b) and then applies an Interior Point Method to solve the nonlinear optimization problem. However, computation of the second derivatives of singular values is computationally difficult. Specifically, in [30], they are obtained through numerical analysis by applying small perturbations to the operating point. Alternatively, since (2) is a semidefinite constraint, we could use semidefinite programming (SDP) by applying a semidefinite relaxation of the AC power flow equations [31], [32]. However, if the relaxation is not tight at the optimal solution, the solution will not be the optimal solution of (1) and, moreover, it will not be feasible.

In this section, we develop a new solution approach that overcomes the drawbacks of the aforementioned approaches. Specifically, our approach uses the first derivatives of singular values obtained using singular value sensitivities, reducing the necessary computation as compared to the second-order method in [30]. We also include the full nonlinear AC power flow equations and solve the resulting optimization problem via an iterative LP algorithm in which 1) the objective function and constraints are linearized such that we can compute a step in the optimal direction using LP, 2) the AC power flow equations are solved for the new operating point (i.e., the original operating point plus the optimal step), and 3) the process is repeated until convergence. Iterative LP [29, p. 371] is commonly used to solve various optimal power flow problems, e.g., [27]–[29], [33].

Our approach is an extension of the iterative NLP approach proposed in [22], which we will now describe. It takes
advantage of the Singular Value Decomposition (SVD) of the Jacobian, i.e.,
\[ J(t) = U(t)\Sigma(t)W(t)^T, \]
where \( \Sigma(t) \) is a diagonal matrix, \( U(t) \) and \( W(t) \) are orthogonal singular vector matrices (i.e., \( U(t)U(t)^T = I \), \( W(t)W(t)^T = I \), and \( u_i(t)^Tw_i(t) = 1 \), where \( u_i(t) \) and \( w_i(t) \) are the left and right singular vectors corresponding to the \( i^{th} \) singular value). Around a given operating point, the approximate SSV of \( J(t) \) is [22]
\[ \tilde{\sigma}_0(t) = u_0(t)^TJ(t)w_0(t), \]
where \( u_0(t) \) and \( w_0(t) \) are the corresponding left and right singular vectors.

Our implicit constraint (1b) can be approximated by (5) and so we can write our problem as a nonlinear optimization problem
\[ \min_{P_e(t), Q_e(t), V(t), \theta(t), \sigma_0(t)} -\alpha\tilde{\sigma}_0(1) + C(P_g(2)) \]
\[ \text{s.t.} \quad \forall t \in T \]
\[ \text{constraints (1c) – (1p), (5)} \]
(6b)
To obtain the solution to our original problem (1), we solve (6), recompute \( u_0(t) \) and \( w_0(t) \) at the new operating point, and repeat the process until convergence. However, the symbolic matrix multiplication in (5) is complex for large systems. Moreover, each iteration requires solving a nonlinear optimization problem. Therefore, the approach does not scale to realistically-sized power systems, as we will show in our case study.

C. New Solution Approach: Iterative Linear Programming using SSV Sensitivities

Our new solution approach uses iterative linear programming where the power flow equations are iteratively linearized around new operating points as in [29, p. 371] and the SSV constraint (5) is linearized using singular value sensitivities. Specifically, the change in the \( i^{th} \) singular value of a generic matrix \( A \) due to a small perturbation in the operating point \( \chi \) is [17]
\[ \Delta \sigma_i \approx \sum_k u_i^T \frac{\partial A}{\partial x^*_k} w_1 \Delta x_k, \]
where \( k \) indexes \( \chi \) and \( \chi^* \) is the current operating point. Therefore, the sensitivity of the SSV of \( J(t) \) is
\[ \Delta \sigma_0 \approx \sum_{n \in \{SV, SVQ\}} u_0^T \frac{\partial J}{\partial \theta_n} w_0 \Delta \theta_n + \sum_{n \in \{SV, SVQ\}} u_0^T \frac{\partial J}{\partial V_n} w_0 \Delta V_n, \]
(8)
where we have suppressed the time dependence of each variable for clarity. Note that our previous work [27], [28] used eigenvalue sensitivities, which required computing \( J^TJ \) and therefore was less scalable.

The resulting linear program solved in each iteration of the iterative LP algorithm is as follows.
\[ \min_{P_e(t), Q_e(t), V(t), \theta(t), \sigma_0(t)} -\alpha\Delta \sigma_0(1) + \sum_{n \in S_G} \frac{\partial C(P_g(2))}{\partial PV_{g,n}(2)} \Delta PV_{g,n}(2) \]
\[ \text{s.t.} \quad \forall t \in T \]
\[ \Delta \sigma_0(t) = \sum_{n \in \{SV, SVQ\}} u_0^T \frac{\partial J}{\partial \theta_n} w_0 \Delta \theta_n(t) \]
\[ + \sum_{n \in \{SV, SVQ\}} u_0^T \frac{\partial J}{\partial V_n} w_0 \Delta V_n(t) \]
\[ f^P_n(\Delta \theta(t), \Delta V(t)) = \Delta PV_{g,n}(t) - \Delta PV_{d,n}(t) \quad \forall n \in N \]
\[ f^Q_n(\Delta \theta(t), \Delta V(t)) = \Delta QV_{g,n}(t) - \Delta QV_{d,n}(t) \quad \forall n \in N \]
\[ \sum_{n \in S_{DR}} \Delta PV_{d,n}(1) = -\epsilon \Delta PV_{loss}(1) \]
\[ N \cap \Delta PV_{d,n}(2) = 0 \quad \forall n \in S_{DR} \]
\[ \Delta PV_{d,n}(t) = \Delta PV_{d,n}(t) \quad \forall n \in N \]
\[ \Delta PV_{d,n}(t) = 0 \quad \forall n \in N \]
\[ \Delta PV_{d,n}(t) = 0 \quad \forall n \in N \]
\[ \Delta \sigma_0(t) \leq \Delta \sigma_0 \]
(9a)
where (9b) is the linearized SSV constraint and (9c)–(9p) correspond to (1c)–(1p), where \( \Delta PV_{loss}(t) = \sum_{n \in N} (\Delta PV_{g,n}(t) - \Delta PV_{d,n}(t)) \) and superscript \( \ast \) denotes the current value of a variable. Constraint (9q) limits the change in \( \Delta \sigma_0(t) \) since the linearizations are only valid near the current operating point.

The solution algorithm is given in Algorithm 1. We initialize the operating points of Periods 1 and 2, \( \chi^*(1), \chi^*(2) \), at the operating point of Period 0, \( \chi(0) \). Then, we compute the constraints of (9) at the current values of the operating points and solve (9) to obtain the optimal change in operating point \( \Delta \chi^*(t) \forall t \in T \). We use those changes to compute updated operating point estimates \( \chi'(t) \forall t \in T \). However, in general, \( \chi'(t) \forall t \in T \) will not be feasible in the AC power flow equations. Therefore, we solve the AC power flow equations for each time period using components of \( \chi'(t) \), specifically, \( P_e, Q_e, Q_d \), and \( V_n \forall n \in S_G \), to obtain the new values of the operating points, \( \chi'(1), \chi'(2) \). We use these values to compute the new values of the SSVs, \( \sigma_0^*(t) \forall t \in T \), and the value of the objective function in (9a). We repeat the process until the absolute value of the objective function in (9a) is less than a threshold (here, \( 10^{-5} \)), and the outputs are the final operating points and SSVs.
Algorithm 1: Iterative LP using SSV Sensitivities

Input: The operating point of Period 0, \( \chi(0) \).

1. \( \chi^*(1) = \chi^*(2) = \chi(0) \)
2. repeat
3. Compute (9b)–(9q) at \( \chi^*(1) \), \( \chi^*(2) \).
4. Solve (9) at \( \chi^*(1) \), \( \chi^*(2) \) to obtain \( \Delta \chi^\text{opt}(t) \forall t \in \mathcal{T} \).
5. \( \chi'(t) = \chi^*(t) + \Delta \chi^\text{opt}(t) \forall t \in \mathcal{T} \)
6. Use \( \chi'(t) \forall t \in \mathcal{T} \) to solve AC power flows to obtain a new \( \chi^*(1) \) and \( \chi^*(2) \).
7. Use \( \chi^*(1) \) and \( \chi^*(2) \) to calculate \( \sigma^*(t) \forall t \in \mathcal{T} \) and the objective function in (9a).
8. until the absolute value of the objective function in (9a) is less than \( 10^{-5} \).

Output: \( \chi(t) \forall t \in \mathcal{T} \), \( \sigma^*(t) \forall t \in \mathcal{T} \)

IV. RESULTS

In this section, we conduct a number of case studies using the IEEE 9- and 118-bus systems. Additionally, we compare the SSV improvement achievable in our base case against that of seven additional cases and compare the performance of our iterative LP (ILP) algorithm against the iterative NLP (INLP) algorithm from [22]. Each iteration of the nonlinear optimization problem (6) is solved with fmincon in MATLAB.

For all case studies, we use the system data from ATPOWER [34] and set \( \Delta \sigma_0 = 0.01 \). We model the entire load at a bus with responsive demand as flexible, i.e., \( 0 \leq P_{d,n} \leq 2P_{d,n}(0) \forall n \in \mathcal{S}_{\text{DR}} \), in order to get a sense for the maximum achievable change in SSV due to DR. In practice, only a fraction of the load at a particular bus will be responsive. We set \( T_1 = 5 \) min and choose \( T_2 \) as the minimum multiple of 5 min that achieves a feasible solution, though in practice \( T_1 \) and \( T_2 \) would be a function of the response time of the generators and the flexibility of the loads.

For the IEEE 9-bus system, we assume the system is initially operating at the optimal power flow solution at \( 5297 \text{/hour} \). A disturbance takes line 4-9 out of service and the SSV drops from 0.2053 to 0.1534. We assume all load is responsive, \( \alpha \) is 10000. If the disturbance is active, we set \( T_2 = 8T_1 = 40 \) min, while if the disturbance is inactive, we set \( T_2 = T_1 = 5 \) min.

For the IEEE 118-bus system, we assume the system is initially operating at the optimal power flow solution at \( 129627 \text{/hour} \). A disturbance takes line 23-24 out of service and the SSV drops from 0.2053 to 0.1534. We assume all load at PQ buses is responsive (1197 MW out a total of 4242 MW of system-wide demand) and set \( \alpha = 10000 \). Whether or not the disturbance is active, \( T_2 = T_1 = 5 \) min.

All computations are implemented in MATLAB on an Intel(R) i5-6600K CPU with 8 GB of RAM.

A. IEEE 9-bus system results

Figures 3 and 4 show the loading pattern, SSV, generation dispatch, and generation cost per hour in each period. The SSV decreases by 25% due to the disturbance and then increases by 7.3% due to the DR actions in Period 1. Again, we show two cases in Period 2 and, again, the SSV is higher and the generation cost is lower if the disturbance is inactive.

B. IEEE 118-bus system results

Figure 6 shows the SSV and generation cost per hour in each period. The SSV decreases by 25% due to the disturbance and then increases by 7.3% due to the DR actions in Period 1. Again, we show two cases in Period 2 and, again, the SSV is higher and the generation cost is lower if the disturbance is inactive.

Figure 7 visualizes the DR actions in Period 1. Red shading in the upper semicircle corresponding to a bus denotes an increase in load, while blue shading in the lower semicircle denotes a decrease in load. The lightning symbol indicates the line removed from service by the disturbance. In this case,
achievability improvement between that obtained in Case 7 and generator actions alone (in these cases, ε is irrelevant because there is no DR). The improvement possible through changes to generator real power generation (Case 3) is slightly greater than that of the base case (6.5% vs. 6.1%), but at a significantly higher generation cost. In Case 4, Generators 2 and 3 are modeled as steam turbine plants with 3 MW/minute (1% of capacity) ramp rates, which reduces their ability to respond and the achievable SSV. Case 5 allows real power generation and voltage magnitudes together (Case 7); however, in practice, generators are ramp limited and so we would expect a realistic achievable improvement between that obtained in Case 7 and Case 8, where we have applied the conservative ramp rate used in Case 4.

The generation costs shown in the table are the costs per hour of Period 1 only; the next subsection describes the cost results of the multiperiod problem. The relative costs of the cases are system dependent; however, assuming that the system is initially dispatched at minimum cost, DR actions will be less expensive than generator actions in Period 1.

We also formulated and solved an optimization problem to determine the minimum load shedding needed to achieve the same SSV improvement as obtained in Case 1 (without system-wide load shedding). We found that the system load would need to drop by at least 17%.

### D. Comparison of Costs

Table II summarizes the cost over one hour of the multiperiod DR strategy (with Period 1 decision variables corresponding to Case 1) for different disturbance restoration times $T_{\text{restored}}$. It also compares the results to the minimum-cost redispatch of generation alone (corresponding to the decision variables in Case 5, i.e., the generators are not limited by ramp rates) to achieve the SSV obtained using DR alone. The cost of each period is computed as the cost per hour times the length of the period, where all periods are 5 min except for the 9-bus system’s Period 2 when the disturbance is active, which is 40 min (as a reminder, this was chosen because it is the shortest multiple of 5 min for which we can obtain a
feasible solution). When $T_{\text{restored}} = 5$ min, the cost per hour of operating the system beyond Periods 1 and 2 but within the hour is equal to the cost per hour of Period 0. However, when $T_{\text{restored}} = 1$ hr, this cost is equal to the cost of using the generators to maintain the SSV achieved in Periods 1 and 2.

As shown in the table, as $T_{\text{restored}}$ increases, the cost of the strategy increases. Comparing the cost of using DR versus generation, we see that the cheaper option is case dependent. In three out of the four cases, DR is cheaper; however, when $T_{\text{restored}} = 1$ hour, generation actions are cheaper than DR actions for the 9-bus system. As described in the previous subsection, DR is always cheaper in Period 1. However, energy payback in Period 2 can be expensive, which is true for the 9-bus system when the disturbance is active, as shown in Fig. 4. Moreover, in this case, Period 2 lasts for 40 min.

Note that the generation costs reported in the table may not be realizable in practice because real generators are ramp-limited. Therefore, in cases in which DR is more expensive then generation, it may still be desirable to deploy DR since generation may not respond in time.

E. Comparison of Algorithms

In this subsection, we compare the performance of the ILP and INLP algorithms. Table III shows the optimal loading pattern and SSV computed using each algorithm for the IEEE 9-bus system considering only Period 1. The solutions/SSVs produced by the algorithms are close. Figure 8 shows the convergence of each algorithm on the 9-bus system considering the full multiperiod problem (disturbance active in Period 2). The solid lines are the results of the ILP algorithm and the dashed lines are the results of the INLP algorithm. The ILP algorithm converges more quickly than the INLP algorithm. Similarly, Fig. 9 shows the convergence of the ILP algorithm on the 118-bus system considering the full multiperiod problem (disturbance active in Period 2). The INLP algorithm does not scale to the 118-bus system.

The computation times are summarized in Table IV. As shown, the ILP algorithm requires significantly less time than the INLP algorithm. The overall computation time is a function of the number of iterations needed and the time required for each iteration, where the former depends on the initial operating point and the maximum step size $\Delta t_0$, and the latter depends on the size of Jacobian matrix. The time could be reduced through 1) parallel computing of the SSV sensitivities, 2) approximating the SSV sensitivity (9b) to only include the system states that most affect the SSV, and/or 3) applying an adaptive maximum step size.

### Table II

<table>
<thead>
<tr>
<th>$T_{\text{restored}}$</th>
<th>Resource</th>
<th>9-bus</th>
<th>118-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 min</td>
<td>DR</td>
<td>5303</td>
<td>129545</td>
</tr>
<tr>
<td></td>
<td>Generation</td>
<td>5360</td>
<td>129905</td>
</tr>
<tr>
<td>1 hour</td>
<td>DR</td>
<td>6441</td>
<td>132777</td>
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<tr>
<td></td>
<td>Generation</td>
<td>6043</td>
<td>132961</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Nominal</th>
<th>Optimal</th>
<th>ILP</th>
<th>INLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{1.5}$ (MW)</td>
<td>90</td>
<td>147.93</td>
<td>149.58</td>
<td></td>
</tr>
<tr>
<td>$P_{1.7}$ (MW)</td>
<td>100</td>
<td>137.23</td>
<td>135.57</td>
<td></td>
</tr>
<tr>
<td>$P_{1.9}$ (MW)</td>
<td>125</td>
<td>29.84</td>
<td>29.85</td>
<td></td>
</tr>
<tr>
<td>SSV</td>
<td>0.4445</td>
<td>0.4715</td>
<td>0.4716</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 8. Convergence of the SSV in Period 1 and the generation cost in Period 2 using the ILP and INLP algorithms for the IEEE 9-bus system.](image)

### V. Conclusion and Future Work

In this paper, we have developed a multiperiod optimal power flow approach to use DR to improve static voltage stability as measured by the smallest singular value of the power flow Jacobian matrix. In addition to formulating the problem, which increases/decreases loads while holding total load constant in a first period and paying back energy to each load in a second period, we have developed an iterative linear programming algorithm using singular value sensitivities. We demonstrated the performance of the approach on the IEEE 9- and 118-bus systems, compared the effectiveness and cost of DR actions to generation actions, and benchmarked our algorithm against an iterative nonlinear programming algorithm from the literature.

Future research will develop formulations that incorporate other stability metrics and will determine how different metrics impact the control of resources. A primary drawback to using the SSV as a voltage stability metric is that it is an indicator of the distance to infeasibility of the power flow equations; it does not contain information about the distance to the engineering or security constraints. Future work will explore and/or develop alternative metrics that do include this information. Other avenues for future work include developing an understanding of why the loading patterns change in the way they do, and improving the computational speed of our algorithm.

### References


TABLE IV
COMPUTATION TIMES (S) | ILP | INLP
--- | --- | ---
IEEE 9-bus system, Period 1 only | 0.4 | 2.5
IEEE 9-bus system, Full multiperiod problem | 1.0 | 6.0
IEEE 118-bus system, Full multiperiod problem | 35 | -